Optics Ch. 2 Homework Sol'ns

2.1 Yellow light, $\lambda = 580\ nm$

piece of paper: 0.003 in $= (0.003\ in)(0.025\ m/in)$
$= 7.5 \times 10^{-5}\ m$

So $n_\lambda = \dfrac{7.5 \times 10^{-5}\ m}{5.8 \times 10^{-7}\ m}$ wavelengths will fit

$n_\lambda \cong 129$

Wavelength of 10 GHz microwaves:

$v = c = \nu \lambda$

$\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}\ m/s}{10^{10}\ Hz} = 3 \times 10^{-2}\ m = 3\ cm$

So, 129 of these will be about 3.9 m

2.2* $c = 3 \times 10^{8}\ m/s$

ν for red ligh $\nu = 5 \times 10^{14}\ Hz$

$\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}\ m/s}{5 \times 10^{14}\ Hz} = 0.6 \times 10^{-6}\ m$

$= 600\ nm$

for 60 Hz e-m wave

$\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}\ m/s}{6 \times 10^{1}\ Hz} = 0.5 \times 10^{7}\ m$

$= 5 \times 10^{6}\ m$

$= 5000\ km$

$(\sim 3000\ mi)$

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2.3* Ultrasonic wave in crystal with

$\lambda = 5\times10^{-5}\,cm = 5\times10^{-7}\,m$

$\nu = 6\times10^{8}\,Hz$

speed $v = \lambda\nu = (5\times10^{-7}m)(6\times10^{8}Hz)$

$= 30\times10^{1} = 300\ m/s$

This is reasonable, though a bit slow for a solid crystal.

2.6 Violin in water (ruined it, I'm sure)

$V = 1498\ m/s \qquad \nu = 440\,Hz$

$$\lambda = \frac{v}{\nu} = \frac{1498\ m/s}{440\ Hz} = 3.42\,m$$

2.12* Transverse Harmonic wave on a string

$v = 1.2\ m/s$

$$y = (0.02m)\sin(157\,m^{-1})x$$

This is a "snapshot" of $\xrightarrow{t=0}$

$$y = A\sin kx - \cancel{\omega t}$$

$\frac{6}{150} = \frac{2}{50} = \frac{4}{100}$ So $k = 157\ m^{-1}$

$A = 0.02\,m$ amplitude

$\lambda = \frac{2\pi}{k} = 4\times10^{-2}m = 4\,cm$ wavelength

$$\nu = \frac{v}{\lambda} = \frac{1.2\ m/s}{4\times10^{-2}m} = \frac{120\ cm/s}{4\,cm} = 30\,Hz$$

period $= \frac{1}{\nu}$

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17* Wave on a string:

$$\psi(x,t) = (30.0\,\text{cm}) \cos\left[\left(6.28\,\tfrac{\text{rad}}{\text{m}}\right)x - \left(20.0\,\tfrac{\text{rad}}{\text{s}}\right)t\right]$$

This is of the form $A \sin(kx \mp \omega t)$ (but a cosine)

a) frequency $\omega = 20.0\,\frac{\text{rad}}{\text{s}} = 2\pi\nu$

$$\nu = \frac{20\ \text{rad/s}}{2\pi\ \text{rad}} = 3.18\ \text{Hz}$$

b) wavelength $k = 6.28\,\frac{\text{rad}}{\text{m}} = \frac{2\pi}{\lambda}$, $\lambda = \frac{2\pi}{k}$

$$\lambda = \frac{2\pi\ \text{rad}}{6.28\,\frac{\text{rad}}{\text{m}}} = 1.00\ \text{m}$$

c) period $= \frac{1}{\nu} = \frac{1}{3.18\,\text{Hz}} = 0.314\ \text{s}$ $\left(\frac{\pi}{10}\right)$

d) amplitude is $30.0\ \text{cm} = A$

e) phase velocity $= \frac{\omega}{k} = \frac{20.0\ \text{rad}\ \text{s}^{-1}}{6.28\ \text{rad}\ \text{m}^{-1}} = 3.18\ \text{m/s}$

f) Since the sign of ωt in $(kx - \omega t)$ is negative, it is propagating toward $+x$ (positive)

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2.18* Show that $\psi(x,t) = A \sin k(x - vt)$ is a sol$^{\underline{n}}$ of wave equation.

Take 2$^{\underline{nd}}$ partials:

$$\frac{\partial \psi}{\partial x} = Ak \cos k(x - vt) \qquad \frac{\partial \psi}{\partial t} = -Akv \cos k(x-vt)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -Ak^2 \sin k(x - vt) = -Ak^2 \psi \qquad \frac{\partial^2 \psi}{\partial t} = -Ak^2v^2 \sin k(x-vt) = -Ak^2v^2\psi$$

Wave eqn is:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$(-Ak^2)\psi = \frac{1}{\cancel{v^2}}(-Ak^2\cancel{v^2})\psi$$

and so it is satisfied

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2-31 Which are traveling waves? $a, b, c > 0$ constants

(a) $\psi(z,t) = (az - bt)^2$ yes, traveling parabola.

you can see this straight from the form, but you could write

$$\psi(z,t) = a^2\left(z - \frac{b}{a}t\right)^2$$

$\uparrow$ v, speed

(b) $\psi(x,t) = (ax + bt + c)^2$

$$= a^2\left(x + \frac{b}{a}t + \frac{c}{a}\right)^2$$

Yes

twice differentiable fn of $x + \frac{b}{a}t$

v, in negative -x direction

(c) Has no time dependence, No.

2-32* (Same as above) [Without graphs

(a) $\psi(y,t) = e^{-(a^2y^2 + b^2t^2 - 2abty)}$

$$= e^{-(ay+bt)^2} = e^{-a^2(y + b/a\, t)^2} \text{ yes}$$

Gaussian  travelling in $-y$

(b) $\psi(z,t) = A \sin(az^2 - bt^2)$

☆ Will need to check w/ wave eqn

see next page

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2-32(b) $\frac{\partial}{\partial x} \sin(az^2 - bt^2)$

$= 2za\cos(az^2 - bt^2)$

$\frac{\partial^2}{\partial x^2} = 2az(-\sin(az^2 - bt^2)) \cdot 2az$

$= -4a^2z^2 \sin(az^2 - bt^2)$

$\frac{\partial}{\partial t} \sin(az^2 - bt^2)$

$= -2bt\cos(az^2 - bt^2)$

$\frac{\partial^2}{\partial t^2} = 4b^2t^2 \sin(az^2 - bt^2)$

This will not have const v, <u>No</u>

(c) $\psi(x,t) = A \sin 2\pi \left(\frac{x}{a} + \frac{t}{b}\right)^2$

could write it

$A \sin 2\pi a^2 (\underbrace{x + \frac{at}{b}}_{x+vt})^2$ <u>yes</u>

propagating negative

(d) $\psi(x,t) = A\cos^2(2\pi(x - t))$

propagating positive

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2-37*

Modulus is $r = |\tilde{z}| = (\tilde{z}\tilde{z}^*)^{1/2}$

$$\tilde{z} = Ae^{i\omega t} \ ; \quad \tilde{z}^* Ae^{-i\omega t}$$

$$|\tilde{z}| = (A^2 e^{i\omega t} e^{-i\omega t})^{1/2} = (A^2 e^0)^{1/2} = (A^2)^{1/2} = A$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$|\tilde{z}| = \left(A(\cos\omega t + i\sin\omega t) \cdot A(\cos\omega t - i\sin\omega t)\right)^{1/2}$$

$$= \left[A^2(\cos^2\omega t - i^2\sin^2\omega t)\right]^{1/2}$$

$$= \left[A^2(\cos^2\omega t + \sin^2\omega t)\right]^{1/2} = \left[A^2 \cdot 1\right]^{1/2} = A$$

Prove that $e^{i\alpha}e^{i\beta} = e^{i(\alpha+\beta)}$

This is simple by properties of exponentials
Could also prove it by
$(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$ and
equating real & im parts, using
trig. identities.