Electromagnetic Scattering Theory

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**Purpose:**

Better understand the nature of electromagnetic scattering using theoretical tools

**Outline:**

- Basic Concepts
- Tools
- Applications
Basics: Electromagnetic Scattering Cross Section

Unpolarized Cross Section

\[ \sigma = \lim_{R \to \infty} 4 \pi R^2 \frac{|E^s|^2}{|E|^2} \]

Polarimetric Cross Sections

\[ \sigma_{ij} = \lim_{R \to \infty} 4 \pi R^2 \frac{|\hat{e}_i \cdot E_s|^2}{|E_j|^2} \]
Basics: Conventional Polarization Basis: H-V

Any polarized EM wave has an electric field vector that can be broken up into horizontal and vertical components.

In practice, field components are measured one at a time. Here the $S_{VH}$ matrix element is measured.
Basics: Radar Scattering Matrix

The Scattering coefficient preserves the phase of the polarized EM wave:

\[ S_{ij} = \sqrt{\sigma_{ij}} \]

General Scattering Equation:

\[ \mathbf{E}^s = \left( \frac{e^{-ikR}}{2 \sqrt{\pi} R} \right) S \cdot \mathbf{E} \]

Scattering Matrix Equation in H-V basis:

\[
\begin{bmatrix}
E_H^s \\
E_V^s
\end{bmatrix} = \left( \frac{e^{-ikR}}{2 \sqrt{\pi} R} \right) \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix}
E_H \\
E_V
\end{bmatrix}
\]

- Scattered Field
- Propagation Term (Normalized Out)
- Complex Sinclair Scattering Matrix
- Incident Field
Basics: Downrange Imaging

FOURIER TRANSFORM

Downrange Image (pixels)
Basics: Crossrange Imaging

FOURIER TRANSFORM

Crossrange Image (pixels)
Basics: Two-Dimensional ISAR Imaging

HH

HV

VH

VV

range (pixels)
crossrange (pixels)

range (pixels)
crossrange (pixels)

range (pixels)
crossrange (pixels)

range (pixels)
crossrange (pixels)

amplitude (dBsm)
Tools

- Analytical solutions for simple objects
- Exact numerical algorithms (low frequency): JRMBOR
- Approximate numerical algorithms (high frequency): Xpatch
- Experimentally measured scattering data
Tools: Analytical Solutions

\[ \frac{\sigma}{\pi a^2} \]

vs.

\[ ka \]
Tools: JRMBOR

- Method of Moments used by JRMBOR
- Small triangular basis functions established along object's surface
- The induced currents function is expanded in terms of the basis functions
- Boundary conditions are applied to determine induced currents
- Scattered electric field is found as an integral over fields due to the currents
Tools: Xpatch

- Xpatch is an approximate yet fast high-frequency EM prediction code

- To predict one ISAR image of a jet at 10 GHz on one workstation:
  - Xpatch: 1 day
  - MoM: 7 billion years

- Xpatch uses ray-tracing (Shooting and Bouncing Ray) to determine scattering
Tools: Xpatch

- Multi-Look Averaging
- CAD Model Formats
- Diffraction
- First Bounce Method
- Physical Optics (PO)
- Shooting and Bouncing Ray (SBR)
- Z-Buffer
Tools: Xpatch

- Number of Multi-Bounces Kept
- Ray Density
- Materials

Multi-Bounce=50
Multi-Bounce=2
Applications

- Predict radar scattering signatures for military intelligence
- Improve the design and use of scattering objects
- Extract physically relevant information from scattering data
- Aid in development of Automatic Target Recognition algorithms
Applications: Improve Design and Use of Objects
Applications: Improve Design and Use of Objects

Pylon: supports objects to be measured

Absorber Sheets: lines anechoic chamber
Applications: Extract physically relevant information

Applications: Extract physically relevant information

Euler Parameter Decomposition:

\[
S_D = U^T S U
\]

\[
U = \begin{bmatrix}
\cos(\tau) & i \sin(\tau) \\
i \sin(\tau) & \cos(\tau)
\end{bmatrix} \begin{bmatrix}
\cos(\psi) & -\sin(\psi) \\
\sin(\psi) & \cos(\psi)
\end{bmatrix}
\]

\[
S_D = \begin{bmatrix}
e^{i\nu} & 0 \\
0 & e^{-i\nu}
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & \tan(\gamma)
\end{bmatrix} m \begin{bmatrix}
1 & 0 \\
0 & \tan(\gamma)
\end{bmatrix} \begin{bmatrix}
e^{i\nu} & 0 \\
0 & e^{-i\nu}
\end{bmatrix}
\]

- \(m\) maximum reflectivity
- \(\psi\) orientation angle
- \(\tau\) symmetry angle
- \(\nu\) bounce angle
- \(\gamma\) polarizability angle
Meaning of the Euler Parameters

- \( m \) (max reflectivity)
- \( \psi \) (orientation)
- \( \gamma \) (bounce)
- \( \gamma \) (polarizability)
- \( \tau \) (symmetry)
Visual Analysis of Euler Parameters: Slicy

ψ orientation (deg)

τ symmetry (deg)

m max magnitude (dBsm)

ν bounce angle (deg)

ψ polarizability (deg)
Automatic Target Recognition Results – H-V vs. Euler

The graph shows the probability of true-positive identification (%) against the probability of false-positive identification (%) for various polarizations and angles. The curves represent different combinations of polarizations and angles, such as HH, VV, VH, HV, and others, each with a distinct shape and position on the graph.
Automatic Target Recognition Results – H-V vs. Euler

[Graph with axes and legend: probability of true-positive identification (%) vs. probability of false-positives identification (%)]