Classification of targets using optimized ISAR Euler imagery

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Outline

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• ISAR Euler Imagery
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Data Acquisition – Full-Polarimetric Linear RCS

- Compact Radar Range: STL
- Target Scale: 1/16th
- RF Bandwidth: 24 Ghz
- Center Frequency: 160 GHz
- Frequency Steps: 128 or 256
- Azimuth Increment: .05°
ISAR Euler Imagery

Euler Parameter Definitions

The known scattering matrix $S$ can be diagonalized to $S_D$ by applying a transform $U$

$$S_D = U^T S U$$

The Euler parameters are defined in terms of $S_D$ and $U$ according to

$S_D = \begin{bmatrix} m e^{i2\nu} & 0 \\ 0 & m \tan^2(\gamma) e^{-i2\nu} \end{bmatrix}$

$U = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & i \sin(\tau) \\ i \sin(\tau) & \cos(\tau) \end{bmatrix}$

$m$ = maximum reflectivity
$\psi$ = orientation angle
$\tau$ = symmetry angle
$\nu$ = odd/even bounce angle
$\gamma$ = polarizability angle
ISAR Euler Imagery

Euler Parameter Derivations

The scattering matrix is reduced to 5 meaningful parameters:

\[ S = e^{i\theta} \begin{bmatrix} a e^{i\beta} & c \\ c & d e^{i\phi} \end{bmatrix} \]

The corresponding Kennaugh power transfer matrix is calculated to be

\[ K = \begin{bmatrix} \frac{1}{2}(a^2 + 2c^2 + d^2) & \frac{1}{2}(a^2 - d^2) & ac \cos b + d \cos f & ac \sin b - cd \sin f \\ \frac{1}{2}(a^2 - d^2) & \frac{1}{2}(a^2 - 2c^2 + d^2) & ac \cos b - cd \cos f & ac \sin b + cd \sin f \\ ac \cos b + cd \cos f & ac \cos b - cd \cos f & c^2 + ad \cos(b - f) & ad \sin(b - f) \\ ac \sin b - cd \sin f & ac \sin b + cd \sin f & ad \sin(b - f) & c^2 - ad \cos(b - f) \end{bmatrix} \]

New variables are defined in terms of the known scattering parameters

\[ K = \begin{bmatrix} A_0 + B_0 & C_\psi & H_\psi & F \\ C_\psi & A_0 + B_\psi & E_\psi & G_\psi \\ H_\psi & E_\psi & A_0 - B_\psi & D_\psi \\ F & G_\psi & D_\psi & -A_0 + B_0 \end{bmatrix} \]
ISAR Euler Imagery

Euler Parameter Derivations

By applying back rotations to the Kennaugh matrix one by one, and requiring stepwise diagonalization, the first three Euler parameters are derived:

\[ \psi = \tan^{-1} \left( \frac{-C_\psi + \sqrt{C_\psi^2 + H_\psi^2}}{H_\psi} \right) \]

\[ \tau = \frac{1}{2} \tan^{-1} \left( \frac{F_\psi}{C_\psi \cos(2\psi) + H_\psi \sin(2\psi)} \right) \]

\[ \nu = \frac{1}{2} \tan^{-1} \left( \frac{B - A_0 + \sqrt{(B - A_0)^2 + (D \cos(2\tau) - E \sin(2\tau))^2}}{D \cos(2\tau) - E \sin(2\tau)} \right) \]

where

\[ B = B_\psi \cos(4\psi) + E_\psi \sin(4\psi) \]

\[ E = E_\psi \cos(4\psi) - B_\psi \sin(4\psi) \]

\[ D = D_\psi \cos(2\psi) - G_\psi \sin(2\psi) \]
ISAR Euler Imagery

Euler Parameter Derivations

• The remaining Kennaugh matrix $K'''$ is independent of everything but $m$ and gamma.

• By matching $K'''$ with its definition below, the final two parameters are derived

\[
K''' = m^2 \begin{bmatrix}
\frac{1}{2} (1 + \tan^4(\gamma)) & \frac{1}{2} (1 - \tan^4(\gamma)) & 0 & 0 \\
\frac{1}{2} (1 - \tan^4(\gamma)) & \frac{1}{2} (1 + \tan^4(\gamma)) & 0 & 0 \\
0 & 0 & \tan^2(\gamma) & 0 \\
0 & 0 & 0 & -\tan^2(\gamma)
\end{bmatrix}
\]

\[
m = \sqrt{A_0 + B_0 + \sqrt{C_\psi^2 + F_\psi^2 + H_\psi^2}}
\]

\[
\gamma = \tan^{-1} \left[ \frac{A_0 + B_0 - \sqrt{C_\psi^2 + H_\psi^2 + F_\psi^2}}{A_0 + B_0 + \sqrt{C_\psi^2 + H_\psi^2 + F_\psi^2}} \right]^{1/4}
\]
Redefining Unavoidable Ambiguities

Unavoidable ambiguities occur when multiple sets of Euler parameters map to the same scattering matrix regardless of how the transform equations are derived.

**Example: spherical scatterer**

\[ \tau = 0^\circ, \nu = 0^\circ, \gamma = 45^\circ, m = 0.5, \psi = \psi_0 \]

\[ S = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \]

- The same scattering matrix results for all values of psi (orientation angle).
- But the orientation angle of a sphere is physically meaningless, thus we can redefine psi in this case to be 0.
- All ambiguities can be redefined and removed in this way.
Redefining Unavoidable Ambiguities

- A tabulation of all possible sets of Euler parameters (to within a degree) and their corresponding scattering matrices leads to identification of all ambiguities
- 41 ambiguities were identified, redefined and removed, a few are shown below

<table>
<thead>
<tr>
<th>Ambiguous Sets</th>
<th>Scattering Matrix</th>
<th>Redefined Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ $\psi$ $\tau$ $\nu$</td>
<td>$\gamma$ $\psi$ $\tau$ $\nu$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 $+$90 0 all</td>
<td>$\begin{bmatrix} 0 &amp; 0 \ 0 &amp; d \end{bmatrix}$</td>
<td>0 90 0 0</td>
<td>wire at 90 deg</td>
</tr>
<tr>
<td>0 0 0 all</td>
<td>$\begin{bmatrix} a &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
<td>0 0 0 0</td>
<td>wire at 0 deg</td>
</tr>
<tr>
<td>45 all 0 0</td>
<td>$\begin{bmatrix} a &amp; 0 \ 0 &amp; a \end{bmatrix}$</td>
<td>45 0 0 0</td>
<td>flatplate, sphere</td>
</tr>
<tr>
<td>0 all $-$45 all</td>
<td>$\begin{bmatrix} -ia &amp; a \ a &amp; ia \end{bmatrix}$</td>
<td>0 0 $-$45 0</td>
<td>positive helix</td>
</tr>
<tr>
<td>0 $-$45 0 all</td>
<td>$\begin{bmatrix} -a &amp; a \ a &amp; -a \end{bmatrix}$</td>
<td>0 $-$45 0 0</td>
<td>wire at $-$45 deg</td>
</tr>
<tr>
<td>0 all 45 all</td>
<td>$\begin{bmatrix} ia &amp; a \ a &amp; -ia \end{bmatrix}$</td>
<td>0 0 45 0</td>
<td>negative helix</td>
</tr>
</tbody>
</table>
Redefining Unavoidable Ambiguities

- With the ambiguities removed, the transform leads to optimized Euler imagery
Characterizing Non-Persistent Scatterers

- Non-persistent scatterers fluctuate rapidly in look angle and degrade accuracy.
- Characterizing non-persistent scatterers should lead to minimizing their effect.
- Non-persistence is thought to be caused by pixels containing multiple scatterers.
- If this is true, it leads to two testable predictions:
  1. The average error in reproducibility should decrease for better resolutions.
  2. The average persistence should increase for better resolutions.
Characterizing Non-Persistent Scatterers

1. The average error in reproducibility should decrease for better resolutions
2. The average persistence should increase for better resolutions
Reproducibility Results: Slicy

![Graph showing error (APD) vs resolution (inch)]

- m
- gamma
- psi
- tau
- nu
Reproducibility Results: Simulator

![Graph showing error (APD) vs. resolution (inch) for different parameters: m, gamma, psi, tau, nu.](image)
Reproducibility Results: T-72M1

Error (APD) vs. Resolution (inch) graph for different parameters.
Persistence Results: Slicy
Persistence Results: Simulator

![Graph showing the relationship between Resolution (inch) and Persistence (deg)]

- m
- gamma
- psi
- tau
- nu
Persistence Results: T-72M1
Conclusion

• Euler transform equations have been explicitly derived using the Kennaugh power transfer matrix

• Optimized Euler imagery has been created using an Euler transform where the ambiguities have been removed

• The reproducibility of Euler images improves for smaller pixel sizes, supporting the multiple-scatterer pixel concept

• The scatterer persistence in look angle improves for smaller pixel sizes, also supporting the multiple-scatterer pixel concept

• Future minimization of the effect of multiple-scatterer pixels should further optimize ISAR Euler imagery for target classification