DESIGN AND ANALYSIS OF AN EULER TRANSFORMATION ALGORITHM APPLIED TO FULL-POLARIMETRIC ISAR IMAGERY

BY

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ABSTRACT OF A DISSERTATION SUBMITTED TO THE FACULTY OF THE DEPARTMENT OF PHYSICS IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN PHYSICS UNIVERSITY OF MASSACHUSETTS LOWELL 2007

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ABSTRACT

Use of an Inverse Synthetic Aperture Radar (ISAR) enables the construction of spatial images of an object's electromagnetic backscattering properties. A set of fully polarimetric ISAR images contains sufficient information to construct the coherent scattering matrix for each resolution cell in the image. A diagonalization of the scattering matrix is equivalent to a transformation to a common basis, which allows the extraction of phenomenological parameters. These phenomenological scattering parameters, referred to as Euler parameters, better quantify the physical scattering properties of the object than the original polarization parameters. The accuracy and meaning of the Euler parameters are shown to be degraded by transform ambiguities as well as by azimuthal nonpersistence. The transform ambiguities are shown to be removed by a case-wise characterization and redefinition of the Euler parameters. The azimuthal nonpersistence is shown to be a result of multiple scattering centers occupying the same cell.

An optimized Euler transformation algorithm is presented that removes transform ambiguities and minimizes the impact of cells containing multiple scattering centers. The accuracy of the algorithm is analyzed by testing its effectiveness in Automatic Target Recognition (ATR) using polarimetric scattering signatures obtained at the University of Massachusetts Lowell Submillimeter-Wave Technology Laboratory and the U.S. Army National Ground Intelligence Center. Finally, a complete ATR algorithm is presented and analyzed which uses the optimized Euler transformation without any previous knowledge and without human intervention. The algorithm is shown to enable successful automatic target recognition.
ACKNOWLEDGEMENTS

The reproducibility and variability of high-resolution Inverse Synthetic Aperture Radar (ISAR) imagery was originally studied as part of a project sponsored and directed by the U.S. Army National Ground Intelligence Center (NGIC) in partnership with the University of Massachusetts Lowell Submillimeter-Wave Technology Laboratory (STL) (1-2). The project included measuring full-polarimetric radar signatures of Main Battle Tanks at Eglin Air Force Base as well as measuring corresponding scaled signatures in submillimeter-wave compact radar ranges. The scaled radar signatures were obtained through the use of exact 1/16th scale model replicas fabricated through the ERADS program and imaged in compact ranges at STL and NGIC. My research efforts to enable successful Automatic Target Recognition through an optimized Euler transform is an extension of the original project. For this reason, I would like to acknowledge the continued sponsorship of NGIC as well as the resources made possible under the ERADS partnership.

I would also like to gratefully acknowledge my wife for her loyal support, my research supervisor, Dr. Robert Giles, for his assistance and guidance, and my coworkers and collaborators Christopher Evans and William Kersey. Lastly, I would like to acknowledge the contributions of the STL staff as well as the physics faculty of the University of Massachusetts Lowell.
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I. INTRODUCTION

A. Scattering and Euler parameter definitions

When an electromagnetic wave is incident on an object, the geometry of the object largely determines the manner in which it scatters the wave. The object's ability to scatter electromagnetic waves can be defined as an effective capture area known as the scattering cross section. The scattering cross section is referred to as the Radar Cross Section (RCS) when incident electromagnetic waves of radio frequencies are used. The RCS is dependent on the angle of incidence and the incident wave frequency, and is equivalent to the projected area of a metal sphere that would scatter the same amount of electromagnetic energy. Most commonly, the radar transmitter and receiver are aligned in a monostatic configuration so that the RCS is an indication of the object's backscattering properties.

The radar cross section $\sigma$ is measured as the ratio of received power $|E_r|^2$ to transmitted power $|E'|^2$, as defined in Eq. (1). The natural reduction of the received power due to spherical propagation in space is normalized out in order to establish an RCS that is independent of the distance $R$ between the scatterer and detector.

$$\sigma = \lim_{R \to \infty} 4\pi R^2 \frac{|E_r|^2}{|E'|^2}$$

Although Eq. (1) defines the RCS in terms of the total electric fields at the detector, polarization information may be extracted by defining polarization-dependent radar cross sections $\sigma_p$ according to Eq. (2).
\[ \sigma_j = \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_i^j}{E_i^j} \right|^2 \] (2)

In Eq. (2), the polarization indices \( i \) and \( j \) can represent either of two polarization bases, such as the traditional horizontal and vertical polarizations. Four independent polarization radar cross sections \( \sigma_j \) are defined so as to span all possible combinations of transmitted polarization \( j \) and received polarization \( i \).

In order to preserve phase information, a more useful complex-valued property known as the scattering matrix element is measured. The scattering matrix element \( S_y \) is defined according to Eq. (3) and is in essence the square-root of the radar cross section.

\[ S_y = \lim_{R \to \infty} \sqrt{4\pi R} \frac{E_i^j}{E_i^j} \] (3)

When the four polarization-dependent scattering elements \( S_y \) are formed into a scattering matrix \( S \), the matrix becomes a complete polarimetric description of the object's scattering properties. The full-polarimetric radar scattering matrix \( S \), often referred to as the Sinclair matrix, describes how the object scatters any transmitted electromagnetic wave \( E' \) to a received wave \( E' \) according to Eq. (4).

\[ E' = \left( \frac{e^{-ikR}}{2\sqrt{\pi R}} \right) S \cdot E' \]

or

\[
\begin{bmatrix}
E'_H \\
E'_V
\end{bmatrix} = \left( \frac{e^{-ikR}}{2\sqrt{\pi R}} \right) \begin{bmatrix}
S_{HH} & S_{HV} \\
S_{VH} & S_{VV}
\end{bmatrix} \begin{bmatrix}
E'_H \\
E'_V
\end{bmatrix}
\]

The term in parentheses in Eq. (4) is simply due to the spherical free-space propagation of a wave, as mentioned previously, and is removed from the scattering matrix \( S \) in order to keep \( S \) independent of the scatterer-receiver distance \( R \). The Sinclair matrix \( S \) is kept

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independent of $R$ in practice by proper calibration of the radar system.

Use of a full-polarimetric Inverse Synthetic Aperture Radar (ISAR) apparatus enables the measurement of the Sinclair matrix, where each element represents the magnitude and phase of the backscattered radiation for a certain polarization of the radar transmitter and receiver. The traditional choice of polarization bases, as presented in Eq. (4), are Horizontal (H) and Vertical (V) polarizations. With this choice of bases, the matrix element $S_{VH}$, for instance, corresponds to the magnitude and phase of the backscattered wave when horizontally-polarized waves are transmitted and vertically-polarized waves are received (see Fig. 1).

![FIG. 1. Schematic representation of the measurement of $S_{VH}$.](image)

When several backscattering measurements are made as the frequency sweeps through a band of values, the object's downrange scattering properties are spectrally assessed. A Fourier transform of the frequency sweep converts this spectral information into a spatial image of the object's downrange scattering properties as shown in Fig. 2.
Similarly, when several scattering measurements are made as the scattering object rotates through a swath of small azimuthal angles, the object's crossrange scattering properties are spectrally sensed. When this swath of scattering values is Fourier-transformed, the spectral information is converted into a spatial image of the object's crossrange radar reflectivity as shown in Fig. 3.
The combination of both types of scattering measurements enables the formation of a two-dimensional spatial image of the object's scattering properties. In this manner, a full-polarimetric ISAR system can construct a two-dimensional spatial grid of resolution cells, where the Sinclair matrix is known for each cell (3).

Traditionally, the phases of the complex-valued Sinclair matrix elements are neglected and their magnitudes are treated separately as displayed in Fig. 4. In this approach, the traditional labels used are HH, HV, VH, and VV, where a label such as HH is equivalent to $|S_{HH}|^2$ imaged on a logarithmic scale.

Theoretically, the Sinclair matrix contains all of the scattering information available about the objects in the resolution cell. However, in the traditional form, the
information is not particularly physically meaningful. Several decomposition techniques have been developed that transform the scattering matrix into more useful parameters (4). Among them, the Euler decomposition transforms the polarization information to phenomenological entities known as Euler parameters, as first developed by Kennaugh and Huynen (5-6). This Euler transform leads to spatial ISAR images of the object's Euler properties.

The foundation of the Euler transform lies in the ability to find an optimal
polarization state through diagonalization of the Sinclair matrix. Once diagonalized, the
dependence of the scattering information on the particular choice of polarization bases as
well as on object symmetry has been removed. Meaningful parameters can then be
extracted from the resulting diagonalized matrix.

A conjugate-similarity unitary transform [see Eq. (5)], is required to diagonalize
the Sinclair matrix $S$ as first shown by Autonne (7). The conjugate-similarity transform,
\[ S_D = U^T S U, \]
must be used because the Sinclair matrix $S$ is defined in the Back Scattering Alignment
(BSA) coordinate system, as opposed to the Forward Scattering Alignment (FSA) used by
the Jones matrix in optical scattering. From the electromagnetic wave's viewpoint, the
BSA used in radar scattering causes a switch of coordinate systems upon scattering as
shown in Fig. 5. Thus, the mathematics of unitary transforms and eigenvalue equations
can be used in the BSA as long as an appropriate conjugation is introduced to account for
the switch of coordinate systems.

FIG. 5. Illustration of the FSA and BSA coordinate systems.
The unitary matrix $U$ is formed from the two conjugate eigenvectors $x$ that satisfy the conjugate eigenvalue equation shown in Eq. (6).

$$Sx = sx^*$$  \hspace{1cm} (6)

Kennaugh interpreted the conjugate eigenvectors $x$ that form $U$ as optimal polarization states or optimal antenna orientations at which the maximum amount of backscattering is attained (5). The Euler parameters are defined in terms of the unitary transform matrix $U$ according to Eq. (7) and in terms of the diagonalized scattering matrix $S_D$ according to Eq. (8).

$$U = \begin{bmatrix} \cos(\psi) \cos(\tau) - i \sin(\psi) \sin(\tau) & -\sin(\psi) \cos(\tau) + i \cos(\psi) \sin(\tau) \\ \sin(\psi) \cos(\tau) + i \cos(\psi) \sin(\tau) & \cos(\psi) \cos(\tau) + i \sin(\psi) \sin(\tau) \end{bmatrix}$$ \hspace{1cm} (7)

$$S_D = \begin{bmatrix} me^{i2\psi} & 0 \\ 0 & m \tan^2(y)e^{-i2\psi} \end{bmatrix}$$ \hspace{1cm} (8)

When representing the Euler definitions in Eqs. (7) and (8) in an expanded form such as in Eqs. (9) and (10), it becomes clear that one parameter, $m$, is a magnitude value, whereas the rest of the parameters are angular rotation variables of different forms.

$$U = \begin{bmatrix} \cos(\psi) & i \sin(\psi) \\ i \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & -\sin(\tau) \\ \sin(\tau) & \cos(\tau) \end{bmatrix}$$ \hspace{1cm} (9)

$$S_D = \begin{bmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \tan(y) \end{bmatrix} m \begin{bmatrix} 1 & 0 \\ 0 & \tan(y) \end{bmatrix} \begin{bmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{bmatrix}$$ \hspace{1cm} (10)

These five variables defined in Eqs. (9) and (10) constitute the Euler parameters. The variable $m$ relates to the maximum reflectivity obtained at the optimal polarization state. The parameter $\psi$ is the orientation angle of the optimal polarization state. Lastly, $\tau$ is the object symmetry angle, $\nu$ is the bounce angle, and $\gamma$ denotes the polarizability angle.
B. Schematic diagrams of the meaning of the Euler parameters

The behavior of the Euler parameters can be represented schematically as in Fig. 6. For example, a long dihedral shape has a maximum reflectivity $m$ when the radiation is polarized in the same direction as the dihedral's axis. The optimal polarization orientation angle $\psi$ is thus measured as the angle between the dihedral's axis and the basis polarization axis. The symmetry angle $\tau$ is the geometric symmetry between the two halves of the object about its axis, which for the dihedral shown in Fig. 6 has a full-symmetry value.

The bounce angle $\nu$ is an indication of whether the electromagnetic wave bounces an odd or an even number of times before backscattering. For the dihedral in Fig. 6, the bounce angle $\nu$ is a double bounce, whereas a flat plate yields a single bounce. The polarizability angle $\gamma$ measures the ability of the scattering object to polarize the reflected radiation. Most scattering objects are nonpolarizing, except for wires and objects approximating wires such as sharp edges.

An intuitive understanding of the Euler parameters can also be gained by examining their possible values as shown in Fig. 7. The polarizability angle $\gamma$ can take on values from fully polarizing at $0^\circ$ to fully nonpolarizing at $45^\circ$. Intermediate values are also possible for partially polarizing objects such as thick wires. The orientation angle $\psi$ assumes values from horizontal orientations at $0^\circ$ to vertical orientations at $90^\circ$ as shown in Fig. 7. Negative orientation values also occur for objects that are oriented counterclockwise from the main axis as opposed to clockwise.

The symmetry angle $\tau$ can range from fully symmetric at $0^\circ$ to fully nonsymmetric at $45^\circ$. The bounce angle $\nu$ goes from odd bounce at $0^\circ$ to even bounce at $45^\circ$. 

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FIG. 6. Schematic representations of the Euler parameters.
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<th>( \gamma )</th>
<th>( 0^\circ ) Polarizing</th>
<th>( 45^\circ ) Nonpolarizing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi )</td>
<td>( 0^\circ ) Horizontal</td>
<td>( \pm 90^\circ ) Vertical</td>
</tr>
<tr>
<td>( \tau )</td>
<td>( 0^\circ ) Symmetric</td>
<td>( \pm 45^\circ ) Nonsymmetric</td>
</tr>
<tr>
<td>( \nu )</td>
<td>( 0^\circ ) Odd Bounce</td>
<td>( \pm 45^\circ ) Even Bounce</td>
</tr>
</tbody>
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FIG. 7. Sample scattering objects that display various Euler parameter values.
The symmetry and bounce angles may take on negative values, but the sign of these parameters has limited meaning. A negative bounce angle has the exact same physical meaning as its positive counterpart to within a phase factor. Often this phase factor indicates only the overall phase which marks the distance from the radar detector to the object. Such is the case with a dihedral which can be measured as having a positive or negative 45° bounce angle depending only on the distance of the detector. Sometimes though, the sign of an Euler parameter's value has more meaning. In the case of helical wires, a symmetry angle value of +45° indicates a nonsymmetrical left-handed helix, whereas the value of -45° indicates a nonsymmetrical right-handed helix.

For completeness, the signs of the Euler parameters are kept intact throughout all of the following mathematical analysis. However, the effect of the signs has been found to be insignificant in practice, and when applied to real data has little impact on the final results. For this reason, only the absolute values of the Euler parameters have been used when real data is concerned.

C. Euler parameter significance and problems

The Euler parameter representation constitutes a set of scattering properties that is more phenomenological than the traditional polarization representations. As such, the Euler parameters better quantify the basic scattering nature of the object, independent from the polarization bases used. The Euler parameters attain significance in the field of object identification for two reasons: First, their phenomenological nature allows an intuitive understanding of how certain objects should appear in an image of scattering properties. Second and more importantly, their phenomenological nature gives the Euler
parameters the potential to improve object identification by isolating the invariant, physical properties of the object.

Successful object identification using radar imaging is crucial primarily in the field of Automatic Target Recognition (ATR) as used in military engagements, but also finds applications in varied fields such as the remote analysis of crop layouts, soil types, and buried-pipe locations (8-11).

The Euler parameters have historically been of limited use, however, because of the presence of ambiguities and their sensitivity to noise (12-13). The ambiguities will be shown as unavoidable, yet removable through a case-wise analysis and redefinition. The parameter's sensitivity and nonpersistence will be shown as partly a result of multiple scattering centers occupying the same resolution cell. An optimized Euler transformation will be presented that removes ambiguities and minimizes azimuthal nonpersistence.

The effectiveness of the optimized Euler representation in ATR will be demonstrated in comparison to the traditional HH-VV representations. The ATR tests involve a controlled, artificial test environment where the look angle is known, as well as a more realistic ATR test environment. As will be demonstrated, the optimized Euler transform overcomes much of the historical Euler parameter limitations and the full ATR algorithm is an improved alternative to traditional target recognition schemes.
II. METHODOLOGY

A. Euler parameter derivations and analysis

Before attempting to overcome the Euler parameter's ambiguities and nonpersistence, the parameters must be explicitly derived in terms of the known Sinclair matrix. The derivation can be pursued by directly inverting the definition equations found in Eqs. (5), (7), and (8), but a more mathematically straightforward approach involves converting the electric-field scattering matrix to a power scattering matrix and then inverting this matrix. After the derivations are complete, a visual analysis of actual scattering images in Euler parameter space allows an initial check of the transform equations.

1. Mathematical derivations in the power representation

Sec. A. of the Introduction provided mathematical definitions of the Euler parameters in terms of the transform matrix $U$ and the diagonalized scattering matrix $S_D$. To be of use in application however, the equations must be inverted to yield transformation equations that transform the measured scattering matrix into the Euler parameters. This is best accomplished by converting to the power matrix representation, and then applying back-rotations in order to stepwise diagonalize the power matrix.

Before analyzing it, the original scattering matrix can be simplified. Scattering that occurs in the backscattered direction can be assumed to be reciprocal, meaning that the crosspolarized radiation will be scattered in the same way such that $S_{HV} = S_{VH}$. This
reduces the meaningful data contained in the scattering matrix to three complex numbers, or six real components. An overall phase factor exists in the matrix that only represents the distance of the source from the scattering object, and thus contains no meaningful information about the object. The overall phase can be safely factored out and ignored. The scattering matrix thus contains only five independent real parameters with meaningful scattering information. For simplicity of notation, these five parameters are labeled $a, b, c, d, f$, with the ignored overall phase labeled $N$, as defined in Eq. (11).

$$S = e^{iN} \begin{bmatrix} a e^{ib} & c \\ c & d e^{if} \end{bmatrix}$$

where $a = |S_{hh}|$, $b = \text{Arg}(S_{hh}) - \text{Arg}(S_{vh})$, $c = |S_{vh}|$, $d = |S_{vv}|$, $f = \text{Arg}(S_{vv}) - \text{Arg}(S_{vh})$, $N = \text{Arg}(S_{vh})$ (11)

To facilitate more straightforward mathematics, the electric-field scattering matrix $S$ in Eq. (11) is then transformed into the power scattering matrix representation. The power matrix $K$ describes how an object scatters a transmitted electromagnetic wave of power $P_i$ to a received wave of power $P_r$ as defined in Eq. (12). As was done with the Sinclair representation, the term in parentheses in Eq. (12) represents the spherical free-space propagation of the wave and is normalized out by proper calibration of the system.

The power scattering matrix representation in Eq. (12) is equivalent to the field scattering matrix representation defined in Eq. (4). In the FSA coordinate system, such as used in optics, the power matrix takes the form of the Stokes matrix. In the BSA coordinate system used in radar scattering, the power matrix is known as the Kennaugh matrix $K$. The Kennaugh power matrix of radar scattering is the same as the Stokes power matrix of optical scattering except for a conjugation of the scattered wave to account for the coordinate system switch of the BSA.
By comparing the definition of the Kennaugh power scattering matrix in Eq. (12) with the definition of the Sinclair field scattering matrix in Eq. (4), it becomes straightforward to derive the relationship of the two representations.

\[
P' = \left( \frac{1}{4\pi R^2} \right) K \cdot P'
\]

or

\[
\begin{bmatrix}
|E_H|^2 + |E_V|^2 \\
|E_H|^2 - |E_V|^2 \\
2 \Re (E_H^* E_V) \\
2 \Im (E_H^* E_V)
\end{bmatrix} = \left( \frac{1}{4\pi R^2} \right) \begin{bmatrix}
k_0 & k_1 & k_2 & k_3 \\
k_4 & k_5 & k_6 & k_7 \\
k_8 & k_9 & k_{10} & k_{11} \\
k_{12} & k_{13} & k_{14} & k_{15}
\end{bmatrix} \begin{bmatrix}
|E_H'|^2 + |E_V'|^2 \\
|E_H'|^2 - |E_V'|^2 \\
2 \Re (E_H'| E_V') \\
2 \Im (E_H'| E_V')
\end{bmatrix}
\]

By expanding out both the Sinclair and Kennaugh matrix definitions in terms of electric fields and matching up coefficients, the Kennaugh matrix \( K \) becomes defined in terms of the known scattering matrix elements according to Eq. (13).

\[
K = \begin{bmatrix}
\frac{1}{2} (a^2 + 2c^2 + d^2) & \frac{1}{2} (a^2 - d^2) & ac \cos b + cd \cos f & ac \sin b - cd \sin f \\
\frac{1}{2} (a^2 - d^2) & \frac{1}{2} (a^2 - 2c^2 + d^2) & ac \cos b - cd \cos f & ac \sin b + cd \sin f \\
ac \cos b + cd \cos f & ac \cos b - cd \cos f & c^2 + ad \cos (b - f) & ad \sin (b - f) \\
ac \sin b - cd \sin f & ac \sin b + cd \sin f & ad \sin (b - f) & c^2 - ad \cos (b - f)
\end{bmatrix}
\]

To simplify notation again, new variables are defined in Eq. (14) in terms of the known parameters according to the pattern first developed by Huynen (14).

\[
K = \begin{bmatrix}
A_0 + B_0 & C & H & F \\
C & A_0 + B & E & G \\
H & E & A_0 - B & D \\
F & G & D & -A_0 + B_0
\end{bmatrix}
\]

The definition of the new parameters in Eq. (14) is found by matching it with the corresponding matrix element in Eq. (13). For example, the Kennaugh parameter \( C \) takes...
on the definition $C = (a^2 - d^2)/2 = (|S_{HH}|^2 - |S_{VV}|^2)/2$.

The dependence of the Kennaugh matrix on each Euler parameter can be removed one at a time by applying back-rotations and then requiring the rotated matrix to be stepwise diagonal as partially outlined by Huynen (14). The dependence of $K$ on the orientation angle $\psi$ is removed first using the rotation matrix defined in Eq. (15).

$$K' = R_\psi^T K R_\psi$$

where

$$R_\psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\psi) & -\sin(2\psi) & 0 \\ 0 & \sin(2\psi) & \cos(2\psi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (15)

The resulting rotated matrix $K'$ has a form, shown in Eq. (16), where new variables are defined in a similar pattern as the previous parameter set to underscore the fact that the Kennaugh matrix still contains the same scattering information, but that each part has been made independent of the orientation angle $\psi$.

$$K' = \begin{bmatrix} A'_0 + B'_0 & C' & H' & F' \\ C' & A'_0 + B' & E' & G' \\ H' & E' & A'_0 - B' & D' \\ F' & G' & D' & -A'_0 + B'_0 \end{bmatrix}$$ (16)

To ensure that the dependence on $\psi$ has been properly removed, we require stepwise diagonalization. The parameter $H'$ must be zero, giving us an equation that can be solved for the orientation angle [see Eq. (17)].

$$\psi = \tan^{-1} \left( \frac{-C + \sqrt{C^2 + H^2}}{H} \right)$$ (17)

The process is again repeated with the rotation matrix shown in Eq. (18) to back-rotate out the dependence of $K'$ on symmetry angle $\tau$. 

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\[ K'' = R_t^T K' R_t \quad \text{where} \quad R_t = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(2\tau) & 0 & -\sin(2\tau) \\
0 & 0 & 1 & 0 \\
0 & \sin(2\tau) & 0 & \cos(2\tau)
\end{bmatrix} \]  

(18)

The resulting matrix \( K'' \) is now independent of both orientation angle \( \psi \) and symmetry angle \( \tau \), and has a form that allows new variables to again be defined [see Eq. (19)] to indicate this independence.

\[ K'' = \begin{bmatrix}
A'' + B'' & C'' & 0 & F'' \\
C'' & A'' + B'' & E'' & G'' \\
0 & E'' & A'' - B'' & D'' \\
F'' & G'' & D'' & -A'' + B''
\end{bmatrix} \]  

(19)

The independence of the Kennaugh matrix \( K'' \) from \( \tau \) means that \( K'' \) has been further partially diagonalized; specifically that \( F'' = 0 \), \( G'' = 0 \), and \( E'' = 0 \). These three equations all contain the same information which is a result of using a sixteen element matrix to represent five independent parameters. Using the equation that results from \( F'' = 0 \) allows us to solve for the symmetry angle \( \tau \) [see Eq. (20)].

\[ \tau = \frac{1}{2} \tan^{-1}\left( \frac{F'}{C'} \right) \]  

(20)

We now back-rotate out the \( \nu \) dependence of \( K'' \) by applying the matrix in Eq. (21).

\[ K''' = R_\nu^T K'' R_\nu \quad \text{where} \quad R_\nu = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \cos(2\nu) & -\sin(2\nu) \\
0 & 0 & \sin(2\nu) & \cos(2\nu)
\end{bmatrix} \]  

(21)

The triply-rotated matrix \( K''' \) is now independent of \( \psi, \tau \), and \( \nu \) and again has a form that allows for the definition of new parameters according to Eq. (22).
\[ K''' = \begin{bmatrix}
    A''' + B_0'' & C'''
    C''' & A_0'' + B''
    0 & 0
    0 & 0
    0 & 0
    D''' & -A_0'' + B_0''
\end{bmatrix} \] (22)

Independence of the Kennaugh matrix \( K''' \) from the bounce angle \( \nu \) requires that \( D''' = 0 \). This requirement allows us to solve for the bounce angle, which results in Eq. (23).

\[ \nu = \frac{1}{2} \tan^{-1} \left( \frac{-(A_0'' - B_0'') + \sqrt{(A_0'' - B_0'')^2 + D''^2}}{D''} \right) \] (23)

The matrix \( K''' \) depends now only on the polarizability angle \( \gamma \) and the maximum reflectivity \( m \). We can easily construct that Kennaugh matrix by taking the Sinclair matrix that depends only on \( m \) and \( \gamma \) and converting it to the power representation, as in Eq. (24).

\[ K''' = m^2 \begin{bmatrix}
    \frac{1}{2} (1 + \tan^4(\gamma)) & \frac{1}{2} (1 - \tan^4(\gamma)) & 0 & 0 \\
    \frac{1}{2} (1 - \tan^4(\gamma)) & \frac{1}{2} (1 + \tan^4(\gamma)) & 0 & 0 \\
    0 & 0 & \tan^2(\gamma) & 0 \\
    0 & 0 & 0 & -\tan^2(\gamma)
\end{bmatrix} \] (24)

By setting \( K''' \) equal to the definition matrix found in Eq. (24), we arrive at a system of equations that can be solved for \( m \) yielding Eq. (25) and for \( \gamma \) as shown in Eq. (26).

\[ m = \sqrt{A_0 + B_0 + \sqrt{C^2 + F^2 + H^2}} \] (25)

\[ y = \tan^{-1} \left[ \frac{A_0 + B_0 - \sqrt{C^2 + F^2 + H^2}}{A_0 + B_0 + \sqrt{C^2 + F^2 + H^2}} \right]^{1/4} \] (26)

Thus, the explicit relations in Eqs. (17), (20), (23), (25), and (26) have been derived that transform the measured Sinclair matrix elements into the Euler parameters. Using these equations, ISAR images of a scattering object can be transformed into spatial images of [Image 0x0 to 610x822]
the object's orientation properties, symmetry, bounce properties, maximum reflectivity, and polarizability.

2. Visual analysis of Euler imagery

The scattering signature of a simple test object known as Slicy was measured in the X-band compact radar range at the University of Massachusetts Submillimeter-Wave Technology Laboratory (STL). The scattering information for Slicy was formed into ISAR images and then transformed into the Euler ISAR images shown in Fig. 8 using the transform equations just derived.

Several noteworthy features provide an initial confirmation and intuitive visualization of how the Euler parameters behave. Slicy contains a trihedral on the top of its base as indicated in Fig. 8(a) that displays the expected characteristics of odd bounce, symmetrical shape, and nonpolarizing behavior.

Two other features, the round cylinder section on the front of Slicy [see Fig. 8(b)] and the flat plate on the front [see Fig. 8(c)] are similar enough to both have an odd bounce, symmetrical shape, and nonpolarizing behavior but differ in orientation angle. The only parts of Slicy that appear as polarizing are the sharp front edge found in Fig. 8(d) which approximates a wire and the back corners in Fig. 8(f) which facilitate diffraction. Even-bounce scattering is properly displayed as only coming from dihedrals such as the long front one in Fig. 8(e) and from multiple object interactions such as between interior sides of the hollow cylinder shown in Fig. 8(g).

Slicy is simple enough that a visual analysis of the Euler parameters is possible and intuitive. More complex and realistic objects such as a T-72 tank, shown in Fig. 9,
have enough scattering interactions to make visual analysis of the Euler parameters difficult.

FIG. 8. The test object Slicy and its associated Euler ISAR images at 0° azimuth and 5° elevation.
FIG. 9. A T-72 M1 tank and its associated Euler ISAR images at 135° azimuth and 5° elevation.

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B. Euler parameter ambiguities

Unavoidable ambiguities arise in the Euler transform as a result of the way the Euler parameters are defined. Using a systematic numerical approach, all of the ambiguities can be identified. Once identified, each ambiguity can be removed through a casewise redefinition of the Euler parameters as appropriate to its physical meaning.

1. The cause of Euler ambiguities

An unavoidable ambiguity arises whenever several different sets of Euler parameters map to the same scattering matrix after applying the Euler definitions found in Eqs. (5), (7), and (8). For instance, the set of Euler parameters \( \gamma=45^\circ, \psi=0^\circ, \tau=0^\circ, \) and \( \nu=0^\circ \) corresponds to a scattering object with the Sinclair matrix shown in Eq. (27).

\[
S = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}
\]  

(27)

However, using the set of Euler parameters \( \gamma=45^\circ, \psi=42^\circ, \tau=0^\circ, \) and \( \nu=0^\circ \) also results in the same Sinclair matrix as in Eq. (27) when applying the Euler definition equations. In fact, this Sinclair matrix will result no matter what value for the orientation angle \( \psi \) is used. Therefore, these Euler parameters are shown to contain an ambiguous orientation angle.

When this scattering object is encountered, the Euler transform equations derived in Eqs. (17), (20), (23), (25), and (26) cannot map the scattering matrix in Eq. (27) to a unique set of Euler parameters. Instead, the transform equations yield a mathematically indeterminate value as shown in Eq. (28).
\[
\psi = \tan^{-1}\left( \frac{-0 + \sqrt{0^2 + 0^2}}{0} \right) \quad \text{for} \quad S = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}
\] (28)

Such an undetermined case in practice yields either software errors or meaningless values dictated by floating-point error.

2. The method for the identification and removal of Euler ambiguities

Using a brute-force numerical approach, all of the possible Euler ambiguities were identified. To accomplish this, all possible sets of Euler parameters to within 1° were tabulated. The corresponding Sinclair matrix for each set in the table was calculated using the Euler definitions found in Eqs. (5), (7), and (8). The entire table was scanned to match identical scattering matrices and thus identify ambiguous cases. It was found that often several different ambiguous cases stem from the same general type of parameter ambiguity and could be grouped into the same class. In this way, 27 classes of Euler ambiguities were found as presented in Table 1.

Once identified, each ambiguous class can be assessed for physical meaning and the Euler parameters redefined accordingly. This is possible because all ambiguities were found to result from a lack of physical significance for certain Euler parameter sets. The ambiguous property can thus safely and arbitrarily be redefined to a consistent value.

Returning to the original example, the Sinclair matrix in Eq. (27) is known to physically correspond to a sphere. A sphere does not physically have an orientation due to the nature of its complete symmetry. Any attempt to define an orientation angle for a perfect sphere would be pointless. The orientation angle \( \psi \) can be safely set to an arbitrary value of 0° for this particular case in order to remove the ambiguity. In this way, all 27
classes of ambiguities were physically analyzed, redefined and removed case by case. A summary of all 27 ambiguities, and their redefined Euler values are found in Table 1.

3. The meaning and redefinition of all ambiguities

All of the possible Euler ambiguities and their redefined values are presented in Table 1. In this table, a term such as "all \( \tau \)" means that the exact same ambiguity exists for all values of \( \tau \) in that special case, whereas a term such as "any \( \tau \)" means that different ambiguities result for different values of \( \tau \) but that for any value of \( \tau \), all the ambiguities belong to the same class and can be handled in exactly the same way.

It should be noted that the maximum reflection parameter \( m \) is an overall multiplicative constant and as such is never ambiguous. Therefore, the parameter \( m \) does not effect the ambiguous nature of the other parameters.
<table>
<thead>
<tr>
<th>Class</th>
<th>Ambiguous Euler Sets</th>
<th>Sinclair Matrix</th>
<th>Redefined Set</th>
<th>Descrip.</th>
</tr>
</thead>
</table>
| 1     | 0 ±90 0 all v        | \[
\begin{bmatrix}
0 & 0 \\
0 & d
\end{bmatrix}
\] | 0 90 0 0 | vertical wire |
|       | 45 -45<all ψ <45 v=ψ |                 |               |          |
|       | 45 -45<all ψ <45 v=ψ |                 |               |          |
|       | 45 all ψ<45 -45 v=ψ+90 | [0 c] | 45 45 0 45 | diplane at 45° |
|       | 45 all ψ<45 -45 v=ψ+90 |                 |               |          |
|       | 45 all ψ>45 45 v=ψ+90 |                 |               |          |
|       | 45 ±90, 0 ±45 0      |                 |               |          |
| 2     | 45 ±45 all τ ±45     |                 |               |          |
| 3     | 0 0 0 all v          | \[
\begin{bmatrix}
a & 0 \\
0 & 0
\end{bmatrix}
\] | 0 0 0 0 | horiz. wire |
| 4     | 45 all ψ 0 0         | [a 0] 45 0 0 0 0 | sphere     |
| 5     | 45 ±90 0 any v<0     | [a e^{ib} 0] 45 0 0 0 | any v>0 retarder |
|       | 45 0 0 any v>0       | [0 a] 45 0 0 0 |          |
|       | 45 -45 any τ<0 0    | b≠0,π          |          |
|       | 45 45 any τ>0 0     |                 |          |
|       | 45 all ψ<0 -45 v=ψ+45|                 |               |          |
|       | 45 all ψ<0 -45 v=ψ+45|                 |               |          |
| 6     | 45 all ψ<0 45 v=ψ-45 | [−a 0] | 45 0 0 45 | horiz. diplane |
|       | 45 all ψ<0 45 v=ψ-45 |                 |               |          |
|       | 45 all ψ>0 45 v=ψ-45|                 |               |          |
|       | 45 ±90, 0 all τ ±45 0|                 |               |          |
| 7     | 0<any γ <45 ±90 0 any v | \[
\begin{bmatrix}
a d^{ib} & 0 \\
0 & d
\end{bmatrix}
\] | 0<any γ <45 90 0 | any v ellipsoid retarder |
|       | 0<any γ <45 ±90 0 any v |                 |               |          |
| 8     | 0<any γ <45 0 0 ±45 0 | \[
\begin{bmatrix}
-a 0 \\
0 & d
\end{bmatrix}
\] | 0<any γ <45 0 0 45 | curved horiz. diplane |
|       | 0<any γ <45 0 0 ±45 0 |                 |               |          |
| 9     | 0 any ψ= ±45 any τ all v | \[
\begin{bmatrix}
a e^{ib} & a \\
0 & a e^{-ib}
\end{bmatrix}
\] | 0 any ψ= ±45 any τ 0 |          |

**TABLE 1.** All possible sets of Euler ambiguities and their redefined values.
<table>
<thead>
<tr>
<th>Class</th>
<th>Sinclaire Matrix Redefined Set</th>
<th>Descrip.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma (\text{deg})$ $\psi (\text{deg})$ $\tau (\text{deg})$ $\nu (\text{deg})$</td>
<td>$\gamma (\text{deg})$ $\psi (\text{deg})$ $\tau (\text{deg})$ $\nu (\text{deg})$</td>
</tr>
<tr>
<td>45</td>
<td>$-90&lt;\psi&lt;45$ or $0&lt;\psi&lt;45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$45&lt;\psi \leq -45$ any $\psi \leq \tau$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>10</td>
<td>$45&lt;\psi \leq -45$ any $\psi \leq \nu$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$135&lt;\psi \leq 90$ any $\psi \leq \nu$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$-45&lt;\psi \leq 45$ any $\psi \leq \nu$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$0&lt;\psi \leq -90$ any $\psi \leq \nu$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$\pm 90$ any $\tau&lt;0$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$0$ any $\tau&lt;0$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$-45$ any $\nu&lt;0$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>45</td>
<td>$45$ any $\nu&gt;0$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>0&lt;any $\gamma&lt;45$</td>
<td>any $\psi \leq -45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>12</td>
<td>$0&lt;\gamma&lt;45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>13</td>
<td>$\pm 90$ any $\gamma&lt;45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>13</td>
<td>$\pm 90$ any $\gamma&lt;45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>14</td>
<td>$0&lt;\gamma&lt;45$ any $\nu=\pm 45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
<tr>
<td>15</td>
<td>$0&lt;\gamma&lt;45$ any $\nu=\pm 45$</td>
<td>$45$ any $\psi$ $45$ $0$</td>
</tr>
</tbody>
</table>

**TABLE 1 (continued).**

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<table>
<thead>
<tr>
<th>Class</th>
<th>Ambiguous Euler Sets</th>
<th>Sinclair Matrix</th>
<th>Redefined Set</th>
<th>Descrip.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \gamma (\text{deg}) )</td>
<td>( \psi (\text{deg}) )</td>
<td>( \tau (\text{deg}) )</td>
<td>( \nu (\text{deg}) )</td>
</tr>
<tr>
<td>16</td>
<td>45 ±90 any ( \tau ) any ( \psi )</td>
<td>[ ae^{\pm i \beta} \begin{bmatrix} c &amp; -ae^{-i \beta} \ e^{-i \beta} &amp; c \end{bmatrix} ]</td>
<td>45 90 any ( \tau ) any ( \psi )</td>
<td>±45,0</td>
</tr>
<tr>
<td></td>
<td>( \nu \leq 45 )</td>
<td>( \gamma \leq 45 ) any ( \psi )</td>
<td>0&lt;( \nu ) any ( \psi )</td>
<td>( \pm 0, \pm \alpha/2 )</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>17</td>
<td>45 ±90 any ( \nu ) ( \nu \leq 45 )</td>
<td>[ -a \begin{bmatrix} c \ a \end{bmatrix} ]</td>
<td>45 ( \nu \leq 45 ) any ( \psi )</td>
<td>0&lt;( \nu ) any ( \psi ) ( \pm 45 )</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>18</td>
<td>0&lt;( \nu ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>[ \begin{bmatrix} ia \ c \end{bmatrix} \begin{bmatrix} c &amp; -ia \ -ia &amp; c \end{bmatrix} ]</td>
<td>0&lt;( \nu ) any ( \psi ) ( \pm 45 )</td>
<td>0 45 0</td>
</tr>
<tr>
<td></td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
</tr>
<tr>
<td>19</td>
<td>0&lt;( \nu ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>[ \begin{bmatrix} ia \ c \end{bmatrix} \begin{bmatrix} c &amp; -ia \ -ia &amp; c \end{bmatrix} ]</td>
<td>0&lt;( \nu ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>0 45 45</td>
</tr>
<tr>
<td></td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>( \nu \leq 45 ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
</tr>
<tr>
<td>20</td>
<td>0&lt;( \nu ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>[ \pm a \begin{bmatrix} c \ c \end{bmatrix} \begin{bmatrix} c &amp; \pm d \ \pm d &amp; c \end{bmatrix} ]</td>
<td>0&lt;( \nu ) any ( \psi ) ( \nu \leq 45 ) any ( \psi )</td>
<td>0 45 45</td>
</tr>
<tr>
<td></td>
<td>45/2&lt;( \psi ) any ( \tau ) ( \nu \leq 45 ) any ( \tau )</td>
<td>45/2&lt;( \psi ) any ( \tau ) ( \nu \leq 45 ) any ( \tau )</td>
<td>45/2&lt;( \psi ) any ( \tau ) ( \nu \leq 45 ) any ( \tau )</td>
<td>45/2&lt;( \psi ) any ( \tau ) ( \nu \leq 45 ) any ( \tau )</td>
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TABLE 1 (continued).
29

<table>
<thead>
<tr>
<th>Class</th>
<th>Ambiguous Euler Sets</th>
<th>Sinclair Matrix</th>
<th>Redefined Set</th>
<th>Descrip.</th>
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<tbody>
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<td>Y (deg)</td>
<td>T (deg)</td>
<td>T (deg)</td>
</tr>
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<td>[i a c]</td>
<td>[c d -id]</td>
<td>0&lt;any y &lt;45</td>
</tr>
<tr>
<td></td>
<td>a&lt;90 and a&lt;e</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0&lt;any y &lt;45 0&lt;any T &lt;45 ±45</td>
<td>[i a c]</td>
<td>[c d -id]</td>
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<td></td>
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<td>[c d -id]</td>
<td>0&lt;any y &lt;45</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0 any y≠±45, any T ≤&lt;45 all v</td>
<td></td>
<td></td>
<td>0&lt;any y≠±45, any T ≤&lt;45</td>
</tr>
<tr>
<td></td>
<td>a≤&lt;45, d/e-b</td>
<td>d/e-d</td>
<td>a&lt;90</td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>±45</td>
<td>±45,0</td>
<td>±45,0</td>
<td></td>
</tr>
</tbody>
</table>

* where \(d = \frac{-a \cos(b) \cos(f)}{a \cos(b) \cos(f)} \) and \(a<90\)

** where \(c = \sqrt{\frac{(a^2 - d^2) a \sin(b - f)}{2 (a \cos(b) + d \cos(f)) (a \sin(b) + d \sin(f))}}\)

and \(0 < 2 c^2 (2 c^2 - a^2 - d^2) (a \cos(b) + d \cos(f))^3 - (c^2 + a d \cos(b - f))^3 (a^2 - d^2)^2 + 4 c^6 (a^2 \cos^2(b) - d^2 \cos^2(f)) (a^2 - d^2)\)

TABLE 1 (continued).

C. Identifying the cause of Euler parameter nonpersistence

1. Multiple-scatterer cells as the cause of nonpersistence

Even after an optimized Euler transform has been created through the removal of all ambiguities, the accuracy of the Euler transform process is still degraded by nonpersistent pixels. Image cells that have rapidly fluctuating Euler parameter values for small changes in azimuthal look angle are considered nonpersistent and lead to parameter degradation. The nonpersistence of scatterer pixels is hypothesized to be the result of
multiple scattering objects occupying the same resolution cell. When multiple scattering centers exist in a close enough arrangement, they become registered in the same resolution cell. The scatterers are measured as if they are in the same physical location, even though they are not. Because the measured scattering values of the cell are a complex sum of all of the scattering center's values within the cell, the location disparity leads to sensitive phase differences that cause the scattering measurements to be nonpersistent as illustrated in Fig. 10. The phase of each scattering center's backscattered wave in Fig. 10 is represented by the length of the arrow.

![Schematic representation of the multiple-scatter-cell hypothesis.](image)

FIG. 10. Schematic representation of the multiple-scatter-cell hypothesis.

The traditional parameters spaces, HH, HV, VH, VV, and even the Euler parameter $m$ are all magnitude variables and as such should be less sensitive to rapidly fluctuating phases in the pixel's complex sum. The remaining Euler parameters, however, are angular variables and are highly dependent on the cell's nonpersistent phases. This
nonpersistence of the Euler parameters for multiple-scatterer cells is therefore hypothesized to degrade the accuracy of the Euler transform process. The accuracy degradation, in turn, is thought to decrease the reproducibility and meaning of scattering measurements as represented in Euler ISAR imagery. The multiple-scatterer-cell hypothesis should thus be tested by determining the relationship between multiple-scatterer cells and parameter accuracy. The characterization of nonpersistence should then lead to better optimization of the Euler transform.

Every scattering object has a natural limit to its persistence due to its size and shape, which acts in addition to the nonpersistence caused by multiple-scatterer cells. Fortunately, though both are wavelength dependent, natural non-persistence and multiple-scatterer-cell nonpersistence operate on different scales and can thus be treated independently. The natural limit of persistence is usually on the order of several degrees, but depends on the wavelength, whereas multiple-scatterer cells cause nonpersistence to within fractions of a degree.

As a visual example and initial verification of both concepts, experimental data was obtained and processed for a pixel containing a single flat plate at normal incidence as well as one pixel containing a trihedral/top hat combination. When the Euler polarizability angle is plotted as a function of azimuthal look angle as shown in Fig. 11, the nonpersistence becomes evident. The first pixel, shown in Fig. 11(a), contains only a flat plate and experiences no multiple-scatter-cell degradation. As a result, the flat plate's polarizability is persistent from approximately -2° azimuthal look angle to 2°. Beyond 2° away from normal incidence, the flat plate's polarizability degrades into fluctuating noise values, and therefore the flat plate becomes naturally nonpersistent outside the 4°
azimuthal span.

In contrast, the second pixel measured contained both a trihedral and top hat shape and was effected by multiple-scatterer-cell degradation. As can be seen in Fig. 11(b),

![Graphical representation of natural and multiple-scatterer-cell nonpersistence](image)

**FIG. 11.** Experimental evidence of natural and multiple-scatterer-cell nonpersistence.

multiple-scatterer-cell nonpersistence occurs on the sub-degree scale. Between 0° and 1° azimuth, the trihedral/top hat pixel's polarizability fluctuates from 25° up to 45°, and then down to 20°.

It should also be noted that valid scattering measurements effected by multiple-scatterer-cell nonpersistence, such as in Fig. 11(b), are distinctly more regular than the fluctuating noise signal occurring in the absence of valid scattering, such as at the graphical extremes of Fig. 11(a).

The multiple-scatterer-cell hypothesis leads to two testable predictions: First, separating the scatterers into different cells should improve image reproducibility. Second, separating the scatterers into different cells should improve azimuthal span.
persistence. As illustrated schematically in Fig. 12, increasing the imaging resolution should achieve the scatterer separation. Therefore, increasing resolution should reduce reproducibility error and improve azimuthal persistence if the multiple-scatterer-cell hypothesis is correct. The predicted trends take the form depicted in Fig. 13. The error-vs-resolution and persistence-vs-resolution trends need to be established for several various scattering objects in order to fully test the hypothesis.

FIG. 12. Schematic representation of the dependence of persistence on resolution.
2. The method of determining error-vs-resolution trends

The multiple-scatterer-cell hypothesis was first tested using a numerically simulated scattering object in order to control all variables, and then using three real objects of varying complexity, whose scattering signatures were measured in a compact range.

The purpose of the tests using the numerically simulated object was to allow for the isolation of the only variables of interest: image reproducibility error and persistence. The approach allowed for the removal of all forms of noise and error typically encountered except for the one form believed to most affect cells with multiple scatterers: minute error in azimuthal look angle.

The numerical simulation involved a simple random distribution of point scatterers with random magnitude and phases. Once this arbitrary scattering object was generated, it was imaged at a desired resolution. Imaging occurs by adding the phase to each scatterer that comes from its distance to the radar, and then complex summing all of
the scatterers contained in the same pixel. Lastly, each pixel is converted to magnitude power space on a logarithmic scale in order to obtain a typical magnitude radar image.

The complex pixel values in the simulation were also converted to obtain a typical angular parameter radar image using a generalized angular transform similar to Eqs. (17), (20), (23), and (26). In this way, the simulation allowed for the abstraction to a generic magnitude parameter corresponding to the real magnitude parameters HH, HV, VH, VV, and m, and to a generic angular parameter corresponding to the real Euler angular parameters γ, ψ, τ, φ.

To obtain a second set of radar images of the same simulated object, a slight azimuthal look angle error of less than a degree was applied and the same imaging process was repeated. Since the same numerically-simulated object was processed twice using the same image formation method, the multiple-scattering cell phenomenon could be isolated. The two image sets were compared to find the reproducibility error caused by multiple-scatter cells, and the whole process was repeated several times at several resolutions to obtain the error-vs-resolution trends for the simulated object.

The reproducibility error was found by comparing the two images pixel to pixel. The pixel's error was measured as the difference of the pixels' values divided by the sum of the values in order to obtain the percent difference $D$ as shown in Eq. (29).

$$D = \frac{|x_2 - x_1|}{x_2 + x_1} \times 100\% \tag{29}$$

Averaging over all of the pixels in the image as well as over all images in the 360° azimuth sweep yields the Average Percent Difference (APD) between the two objects. In this case, the APD represents the reproducibility error because the two images are of the
same scattering object. The validity of using the APD method to determine the correlation between radar images has been established by Giles (15).

It should be noted that in the APD method, the magnitude parameters are thresholded and shifted to all positive values relative to a noise floor before the APD can be computed (15). For angular parameters, the pixel values can be used directly but the denominator of Eq. (29) becomes the maximum possible sum and not the particular pixel sum. After the APD between the simulated object's images was found at several different resolutions, the error-vs-resolution trends could be constructed. The trends for the simulated object are found in the Results section.

The error-vs-resolution trend for each real object was determined in the same way as for the simulated object; using the APD of two image sets. The main difference was of course that the real objects had to be measured in physical radar ranges, and the data was then transformed into ISAR images. The other difference was that the real-object error-vs-resolution trends were obtained for the actual parameters HH, HV, VH, VV, m, γ, ψ, τ, and v whereas the simulated-object error-vs-resolution trends were obtained for the generic magnitude parameter and the generic angular parameter.

The three real objects used in this analysis were chosen to ensure that the error-vs-resolution trends are not dependent on object shape or object complexity. The first object, Slicy, represents a simple set of scattering elements as shown in Fig. 14. The Simulator, displayed in Fig. 15, was chosen next to give a middle level of complexity. The last object investigated was a T-72 M1 which displays great complexity as can be seen in Fig. 16. All three scattering objects were measured as 1/16th scale replicas produced through the ERADS program. The radar scattering signatures were obtained in the 160
GHz compact radar range at the University of Massachusetts Lowell Submillimeter Technology Laboratory (STL) to match a full-scale radar used at 10 GHz (16). The object's material properties were also scaled to give the same reflection and transmission behavior as their full-scale equivalents, as developed extensively at STL (17). Each model was measured in free space (no ground plane) to determine the scattering phenomena independent of environmental effects. The 160 GHz compact radar range at STL was operated with a 24 GHz bandwidth, at a fixed elevation angle of 5.00°, and with 0.05° azimuth angle increments stepping through the entire 360° azimuth circle. Imaging each radar signature at 1.00° azimuthal increments yielded 360 images of each object per signature.

To obtain an error-vs-resolution trend of these three objects, each signature was obtained twice at several resolutions. Both measurements were imaged and transformed into Euler ISAR images at each resolution, and the reproducibility error at each resolution was measured as the APD. The resulting error-vs-resolution trends for the simulated object and the three real objects were found to be consistent with the predicted trends, as can be found in Sec. A. of the Results.
3. The method of determining persistence-vs-resolution trends

To obtain a persistence-vs-resolution trend for each object, whether simulated or real, images were obtained at 0.2° look angle increments over the full 360° sweep. Each image was back-rotated in azimuth by the amount of its look angle, as described fully in Sec. C.4 of the Methodology. The image back-rotation forces the object to remain stationary in the sequence of images while the look angle increments. This ensures that each physical scattering element on the object remain in the same spatial location while the look angle sweeps, enabling a persistence value to be determined. This fixing in place of the scattering object works only in the far-field approximation where the radar system is far enough away to receive an essentially flat wave-front, and therefore guarantees rectangular pixel regions. Once this back-rotation is accomplished, each imaging parameter of each pixel has a trend-line as a function of azimuthal look angle (see Fig. 18). This trend-line in azimuth angle exists independently for each Euler parameter and each traditional imaging parameter.

The pixel persistence was measured as the extent along the azimuthal trend-line for which the pixel value stayed within 15% of its original value. To determine this extent, the percent difference $D_\phi$ was measured between the pixel value $x$ at a starting azimuth $\phi_0$ and the same pixel's value at some later azimuth $\phi_n$, according to Eq. (30).

$$D_\phi = \frac{|x(\phi_n) - x(\phi_0)|}{|x(\phi_n) + x(\phi_0)|} \times 100\%$$ (30)

The value of $D_\phi$ was determined for increasing azimuths, $\phi_n = \phi_0 + 0.2^\circ, \phi_0 + 0.4^\circ, \ldots$ until $D_\phi$ became greater than 15%. The azimuth $\phi_N$ where $D_\phi$ exceeded 15% was considered
the point where the scatterer's persistent behavior ends. Thus the azimuthal persistence for a particular scatterer was calculated as \( P_\phi = \phi_N - \phi_0 \).

The overall persistence of an object's signature for a certain imaging parameter at a given resolution is the average of all of the pixel persistence values. When repeated measurements of the overall persistence are taken at many resolutions, the persistence-vs-resolution trend for each parameter and each object can be established. The resulting persistence-vs-resolution trends, as found in the Results section, are quite consistent with the trend predicted by the multiple-scatterer-cell hypothesis.

4. The Exact Back-Rotation method

In order to perform a more exact image back-rotation to enable persistence values to be measured, a novel algorithm was developed. The rotation algorithm takes advantage of the fact that the scattering data can be rotated during image formation. The traditional image rotation algorithm applies a rotation matrix to each pixel's spatial coordinates, then reassigns the pixels values of the already formed image to the new coordinates. This traditional rotation method, however, leads to image degradation. This degradation results from the fact that the new, rotated coordinates are often non-integer while the nature of a discrete, pixelized image allows only integer-valued locations.

Instead of shuffling around the values of already formed pixels, the novel rotation algorithm looks at the frequency-space data and can thus look up the pixel value at the exact non-integer spatial coordinates needed to construct a rotated image.

Conceptually, the formation of an ISAR image using a Fourier transform can be
thought of as adding the appropriate phases to the data in order to steer the synthetic radar antenna's focal point to the pixel of interest. A two-dimensional Fourier transform applied to the whole frequency-azimuth data block is traditionally thought of as steering the radar to a raster-scan of sequential integer pixel locations.

The new back-rotation algorithm uses the same Fourier transform to form the image, but instead of steering to integer locations, it steers to the exact non-integer spatial locations required to form an exact rotated image. In a programming sense, the rotation is applied to an empty grid of pixel locations (see Fig. 17) and then the Fourier transform is shifted to the exact non-integer locations in the grid to form a rotated image.

Mathematically, the traditional approach involves a two-dimensional discrete Fourier transform applied to the azimuth-frequency data \( H(\phi, f) \) in order to form the spatial image \( h(x, y) \), followed by the rotation of the image by an angle \( \theta \) [see Eq. (31)].

\[
h(x, y) = \sum_{\phi=0}^{\Phi_{\max}} \sum_{f=0}^{F_{\max}} H(\phi, f) e^{-i2\pi f y} e^{-i2\pi \phi x} df \ d\phi
\]

followed by

\[
h(x', y') = h(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)
\]

In contrast, the exact back-rotation applies the coordinate rotation first and then applies the two-dimensional discrete Fourier transform to form the image as shown in Eq. (32).

\[
h(x', y') = \sum_{\phi=0}^{\Phi_{\max}} \sum_{f=0}^{F_{\max}} H(\phi, f) e^{-i2\pi f (x \sin \theta + y \cos \theta)} e^{-i2\pi \phi (x \cos \theta - y \sin \theta)} df \ d\phi
\]

Both methods are mathematically equivalent except for the fact that when the rotation is performed last, it must round to the nearest available pixel coordinates, but when the rotation is performed first, no rounding occurs.
D. Minimizing the effects of Euler parameter nonpersistence

1. The persistence-optimized Euler transform

Because the error-vs-resolution and persistence-vs-resolution trends presented later in the Results section confirm that nonpersistence is chiefly caused by multiple-scatterer cells, parameter accuracy should be improved when the effects of the multiple-scatterer cells are minimized. A persistence-optimized Euler transform was developed
which minimizes the multiple-scatterer cells' impact by weighting up the more persistent pixels. The optimization was aimed to improve Euler parameter accuracy and ultimately to improve Automatic Target Recognition (ATR).

The persistence-optimized Euler transform used a radar signature set of 360 images taken at 1° azimuthal increments and exactly back-rotated each image using the Exact Back-Rotation method described in Sec. C.4 of the Methodology. Once the back-rotation was applied, the persistence of each pixel was measured as illustrated in Fig. 18. The percent difference $D_\phi$ between a pixels' value and the value of the same pixel at a later azimuth was measured according to Eq. (30). The azimuth $\phi_N$ at which $D_\phi$ exceeded 12% was found and the persistence was measured as $P_\phi = \phi_N - \phi_0$. A weight was assigned each pixel directly proportional to its persistence $P_\phi$. In this way, the more persistent pixels are given higher weights, being deemed as the more accurate, single-scatterer pixels. Specifically, pixels with a persistence of 1°, 2°, 3°, 4°, or 5° or greater received

![Diagram of azimuthal persistence](file.png)
respectively the weights 0.1, 0.3, 0.6, 0.8, and 1.0 for the magnitude parameters and 0.1, 0.4, 0.7, 0.9, and 1.0 for the angular parameters.

When comparisons were made between images of different objects, each pixel's percent difference $D$ was computed using Eq. (29) and multiplied by its persistence weight. The average of the weighted percent difference was then taken over all pixels in the image and over all images to establish a persistence-weighted APD.

2. The method for testing the effectiveness of persistence optimization

The effect of the Euler transform and persistence optimization on ATR performance was measured first in an artificial, controlled way. By using previous knowledge of the azimuth of the unknown vehicle, as well as by letting a human manually achieve the best thresholds and registration, the effect of persistence optimization on ATR performance was isolated. The tests were later repeated using a realistic, complete ATR algorithm that needs no previous knowledge about the unknown vehicle and no human intervention, as described in Sec. E. of the Methodology.

Both the controlled and the full ATR performance tests were carried out on a suite of similar tanks. The tanks in the test suite were chosen to all be spatially similar, differing mostly in equipment configuration. This test suite represents one of the hardest challenges that an automatic target recognition algorithm could encounter. Such a challenging test suite was chosen because it best reveals the effect of the Euler parameters. The traditional, easier task of differentiating spatially dissimilar vehicles, such as tanks and trucks, can often be achieved using an ATR algorithm based on size alone, without resorting to detailed polarimetric scattering information. Only in the harder
task of differentiating similar tanks differing mostly in equipment can the Euler parameters exhibit their strength in improving ATR. Two of the tanks in the test suite are presented in Fig. 19 and Fig. 20 to demonstrate how similar all of the objects in the test suite are. Only this challenging test suite was used throughout this research. This fact must be remembered in order for the results to make sense when compared with other ATR research efforts.

Each of the tanks in the test suite was measured several times as 1/16\textsuperscript{th} scaled model replicas in the high-fidelity 522 GHz compact radar range at the University of Massachusetts Lowell Submillimeter Technology Laboratory (STL), to give the equivalent of full-scale tanks measured at 32.625 GHz. The use of full-scale field data was omitted in this research. The reason for this is that the signature variability present in field data makes any results derived from the field data much less conclusive regarding the effect of the Euler parameters and persistence optimization on ATR performance. The nature of signature variability in field data as well as in compact range data has been documented elsewhere (1-2).
The scale models used in this research were built and measured through the ERADs program in conjunction with the United States Army's National Ground Intelligence Center (NGIC). The scaled vehicles were built with scaled material properties so that the surface reflection and transmission behavior is equivalent to the full-scale vehicles, as developed at STL (17). Each model was measured in free space without a ground plane in order to isolate the structure's scattering phenomena independent of environmental factors. The 522 GHz compact radar range at STL was operated with a 16.384 GHz bandwidth, at a fixed elevation angle of 5.00°, and with 0.01° azimuth angle increments stepping through the full 360° circle. Forming ISAR images of the scattering data at 1.00° azimuthal increments created sets of 360 images for each radar signature. Each image consisted of 128x128 pixels, where each pixel had a resolution of 0.360 inches in both dimensions, representing a resolution of 5.763 inches per pixel at full scale.

The different tanks have been labeled with the codes CI, CK/CL, and AZ. Thus, for example, two tanks that match and result in a correct "same target" identification are CI to CI, or CK to CL. Two tanks that should not match and thus deserve a "different target" designation are such as CI to CK or CI to AZ. A more detailed description of the datasets used can be found in the Appendix.

In each comparison, a full 360°-azimuth sweep of images was compared pixel to pixel using Eq. (29) and averaged to yield image-to-image Average Percent Differences (APD's). The probability densities were then formed for all the APD's in the azimuth sweep for each comparison. The APD probability density plots allow a quick visual determination of the effectiveness of the Automatic Target Recognition (ATR) algorithm. The smaller the overlap between the same-target APD probability curve and the different-
target APD probability curve, the more effective the ATR method, as shown in Fig. 21.

An alternate and more succinct method of plotting ATR performance results is the Receiver Operating Characteristic (ROC) curve (18). To construct a ROC curve, a decision threshold is scanned across the probability density curves as shown in Fig. 22. At
each possible decision threshold, such as at lines A, B, and C of Fig. 22, all comparisons scores below the threshold are designated as positive identifications. When compared to ground truth, the probability of True-Positive (TP) identifications and False-Positive (FP) identifications are computed for each threshold and plotted as a point on the ROC curve as shown in Fig. 22. As the decision threshold sweeps across the probability curves, several TP-vs-FP points are plotted and define the ROC curve. For simplicity, only one matched and one unmatched comparison are shown in Fig. 22. However, in order for the ROC curve to represent overall ATR performance, typically all possible comparisons are plotted as probability density curves and one ROC curve is constructed from the complete ensemble of comparisons.

Mathematically, the area bounded by a probability density curve $P_D$ and the decision threshold $t$ are represented as integrals. A ROC curve is defined parametrically in terms of the decision threshold $t$ as shown in Eq. (33). For the x-axis, the probability of true-positive identification $P_{TP}$ at threshold $t$ equals the area bounded by the matched-comparisons probability density curve $P_{D1}$ and the threshold $t$. For the y-axis, the probability of false-positive identification $P_{FP}$ at threshold $t$ equals the area bounded by the unmatched-comparisons probability density curve $P_{D2}$ and the threshold $t$.

$$P_{TP}(t) = \int_0^t P_{D1} dt$$

$$P_{FP}(t) = \int_0^t P_{D2} dt$$

(33)

An ATR system that has better target recognition would have a ROC curve that is further up in the top-left corner of the plot, where false-positive identifications are less...
probable and true-positive identifications are more probable (see Fig. 23). Although ROC curves contain the same information as the bell-shaped probability density curves, they have the advantage of allowing several systems to be compared on the same plot.

![ROC Curve Diagram](image)

**FIG. 23.** Schematic diagram of the meaning of ROC curves.

The controlled ATR test was carried out for every tank in the test suite and for the various parameter possibilities. In this way, a large suite of results has been obtained which determines the effect of each Euler parameter, each traditional parameter, and persistence optimization on ATR performance (see the Results section).

**E. Implementation of the full ATR algorithm**

As the final and most realistic test of how the Euler transform and persistence optimization effect target recognition, a complete ATR algorithm was constructed and the test suite of battle tanks applied to it. The full ATR algorithm requires no previous
knowledge of the unknown vehicle such as its azimuth, and needs no human intervention to set thresholds or determine the best image registration. The algorithm requires only enough radar scattering data of the unknown vehicle to form one image and then provides as the output the identification of the unknown vehicle in the image.

To determine the most effective ATR method, the algorithm was developed to be able to use any of the parameters HH, HV, VH, VV, \( m, \gamma, \psi, \tau, \) or \( v \) and with or without persistence optimization turned on. Because the full ATR algorithm was developed to be in the same form as would be used in real ATR situations, the algorithm only inputs one image and outputs one identification. In order to analyze performance using a large sample size, an over-arching, separate module was developed that runs several tests through the full ATR, determines its success and plots the performance curves as found in the Results section.

1. Overview of the full ATR algorithm

The full ATR algorithm takes the one unknown vehicle image and attempts to match it to a library of pre-rendered vehicle images. The unknown vehicle is recognized as the library vehicle to which it best matches. Several steps are involved to properly form the image of the unknown vehicle, identify its azimuth, and compare it to all of the images in the library at that azimuth in order to find a match. The major steps in the full ATR algorithm were formed into separate modules to organize the process flow as presented in Fig. 24.

Before the full ATR algorithm can be used, the library of reference images must be rendered. To create the library, each object of interest is measured in a compact radar
range and formed into ISAR and Euler ISAR images at every azimuth to within 1°. A more complete description of how the library is generated is found in the Sec. E.2 of the Methodology.

As shown in Fig. 24, first the scattering data of the unknown vehicle is used to find an approximate center of the structure. The center-finding module functions by creating a temporary image of the unknown vehicle using a two-dimensional FFT, and then calculates the vehicle's center of reflectivity power as its approximate geometric center.

Next, the azimuth-finding module takes the scattering data and forms a temporary image, then attempts to determine the azimuthal orientation of the unknown vehicle as described in Sec. E.3 of the Methodology. Because the module cannot identify the azimuth to a greater accuracy than a few degrees, a handful of possible azimuths is reported. At each possible azimuth, as shown in Fig. 24, a separate back-rotated image is formed and compared to the library images. Whichever possible azimuth best matches the library images is considered the true azimuth.

Once an approximate center and a possible azimuth is given, the scattering data is formed into a back-rotated image using the Exact Back-Rotation Method described previously in Fig. 17. Following the process flow in Fig. 24, the non-ambiguous Euler transform is next applied to the scattering matrix image to yield Euler ISAR images, and preset thresholds are applied to eliminate pixels that are purely noise.

Each of the vehicle's images at each possible azimuth is compared to all of the reference vehicles in the library at its azimuth. To make the comparison, two images are first registered, or aligned, using an autocorrelation and shifting technique. Then the
FIG. 24. Schematic process flow of the full ATR algorithm.
percent difference $D$ already defined in Eq. (29) is found, with or without the persistence weighting applied, depending on the test being run. The percent differences are averaged to yield the APD, and all of the APD scores for all the comparisons are tabulated. The lowest APD among all of the tabulated comparisons is found in order to identify the best match, and thus assign the proper identification of the unknown object. Several of the modules are complex enough to warrant a deeper description.

2. Object library creation and utilization

The pre-rendering of the library must be executed as part of the ATR system deployment process. To identify structures as targets, each structure of interest that could be encountered in the given operational scenario must be added to the reference library. Structures that are not important to the scenario are omitted from the library. When encountered in the field, the ATR algorithm would identify such structures as "unknown". To add each object of interest to the library, each is placed in a compact radar range or outdoor radar range, and its radar signature is measured for the full azimuth sweep to enable the formation of images at $1^\circ$ azimuth increments. The operational parameters, such as center frequency and resolution, of the radar range used to build the library must match the operational parameters of the radar system used in the field. For the particular test suite of battle tanks used in this study, a 522 GHz compact range was used with $1/16$th scale replicas imaged at a $5.00^\circ$ elevation angle to match a potential 32.625 GHz radar detector in the field, as described previously.

Once the signatures of all the library objects of interest are obtained, the back-rotated, centered, Euler transformed, thresholded images are formed in the same way that
the image of the unknown object is formed. Unlike the unknown object though, the library objects are used as the basis from which to determine the persistence weights, as presented previously in Fig. 18.

3. The Azimuth Identification method

The role of the Azimuth Identification module is to take the image of the unknown vehicle and determine the azimuthal angle between the vehicle's orientation and the radar detector's orientation. Because the scattering behavior and ISAR image of any object is highly dependent on its azimuthal orientation, the azimuth must be determined before meaningful comparisons can be made between the unknown image and library images. In order to accommodate the fact that the azimuth can only be determined within a few degrees of accuracy, this module outputs a few possible azimuths to be tested, instead of just one.

It should be noted that the ISAR image of an object is also highly dependent on its elevation angle relative to the radar detector, but this study assumes the elevation angle to be constant. The unknown vehicle can be assumed to always lay flat on a flat landscape. This assumption leads to the fact that the elevation angle is a constant determined by the position of the radar detector, and can be factored in at the stage of library creation. There may be cases where the elevation angle assumptions are not accurate, but such cases are beyond the scope of this research.

The Azimuth Identification module operates by assuming the vehicle to be a rectangle and then finding the angle at which the short side of the rectangle is parallel to the images' x-axis. First, the image is thresholded so that each pixel is set to either 0 to
designate an empty pixel or to 1 to designate that it is part of the vehicle. This thresholding allows the vehicle details to be removed and the vehicle's rectangular outline to be defined. This flat image is then rotated through several angles until the angle is found that brings the vehicle's front leading edge parallel to the bottom of the image. At each angle of rotation, the image is summed vertically to collapse the vehicle to a one-dimensional profile along the x-axis. The image rotation angle at which this box-shaped profile becomes narrowest is then identified as the angle at which the vehicle has been rotated to bring the rectangular vehicle to face towards the image bottom, and thus is the negative of the azimuthal look angle.

4. Image comparison and target recognition

Before two ISAR images can be compared and an Average Percent Difference (APD) score determined, the objects must be properly registered. Any spatial misalignment of the two objects will greatly inflate the APD score away from its true value. Two steps are taken to register the images: First, the autocorrelation of the two images is taken to determine the shift for which the two images best match and the images are shifted accordingly. Second, minor shifting is used to ensure exact registration. One image is given a few possible shifts of a few pixels, the APD is measured and then the shift with the lowest APD is retained.

To determine the difference between vehicles for each comparison, the standard APD is used, with persistence weighting turned on or off. As the final step, the full ATR algorithm simply finds the comparison with the lowest APD score and signifies that library object as the best match. The ATR Algorithm thus outputs the object name, type,
5. The method for testing the performance of the full ATR algorithm

For the purpose of performance analysis, a separate module was developed that encapsulates the full ATR algorithm and runs through hundreds of test azimuths and all of the test vehicles. This module is provided with the ground truth of each vehicle's identity so that it can determine whether the full ATR algorithm has succeeded or failed to recognize the vehicle. This module constructs the standard ATR probability density curves to allow visualization of the full ATR algorithm's performance. The testing module also forms Receiver Operator Characteristic (ROC) curves as a more succinct summary of ATR performance. The suite of battle tanks already discussed was used to test the full ATR algorithm, and the resultant plots that this module generated are presented in the Results section below.
III. RESULTS

A. Euler resolution trends and the cause of nonpersistence

The error-vs-resolution and persistence-vs-resolution trends are presented for the simulated object and the three real objects. In each case, the trends match those predicted and confirm the multiple-scatterer-cell-hypothesis as the chief cause of nonpersistence.

1. Numerically simulated resolution trends

The error-vs-resolution plot of the numerically simulated object (see Fig. 25) shows a trend consistent with prediction. The generic magnitude parameter in Fig. 25(a) has the expected flat trend and is thus insensitive to the number of scatterers in a given cell. The angular parameter shown in Fig. 25(b), however, experienced the expected

![Figure 25](image-url)
increase in reproducibility error for worse resolutions (larger pixel sizes) and thus is
sensitive to the number of scatterers in each pixel. It must be emphasized again that the
use of a numerically simulated scattering object allowed the complete removal of all
forms of noise except minor look angle error, thus the resultant trends are genuinely a
result of multiple-scatterer cell phenomena. To verify further that no other noise was
present, the resolution was increased until there were no pixels containing multiple
scatterers. As a result, the reproducibility error became a perfect zero no matter the
amount of look angle error.

The persistence-vs-resolution trends of the simulated object (see Fig. 26) also
match the expected trends. The magnitude parameter shown in Fig. 26(a) is flat as
expected and thus does not experience nonpersistence as a result of multiple-scatterer
pixels. The angular parameter shown in Fig. 26(b) shows the expected decrease in
azimuthal persistence for worse resolutions. With satisfactory trends resulting from
simulated data, the results from the real scattering objects can now be analyzed.

![Graph](image)

**FIG. 26.** Numerically simulated persistence-vs-resolution trends for 0.6° look angle error.

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2. Measured error-vs-resolution trends

The measured error-vs-resolution trends found in Figs. 27-32 are consistent with the trends predicted by the multiple-scatterer-cell hypothesis.

![Graph showing error-vs-resolution trends for Euler parameters](image1)

*FIG. 27. Slicy's error-vs-resolution trends for the Euler parameters.*

![Graph showing error-vs-resolution trends for HH-VV parameters](image2)

*FIG. 28. Slicy's error-vs-resolution trends for the HH-VV parameters.*

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FIG. 29. Simulator's error-vs-resolution trends for the Euler parameters.

FIG. 30. Simulator's error-vs-resolution trends for the HH-VV parameters.

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FIG. 31. T-72's error-vs-resolution trends for the Euler parameters.

FIG. 32. T-72's error-vs-resolution trends for the HH-VV parameters.
The angular Euler parameters, $\gamma$, $\psi$, $\tau$, and $\phi$, showed the expected trends of higher reproducibility error with worse resolutions for all three objects (see Figs. 27, 29, and 31). As predicted, every angular parameter of every scattering object is degraded by poorer resolutions and thus by more cells containing multiple scatterers.

Likewise, every magnitude parameter, including the Euler parameter $m$ as well as the traditional HH, HV, VH, and VV parameters showed the expected flat trend (see Figs. 28, 30, and 32). This verifies that the magnitude parameters are not sensitive to phase differences, and thus do not have multiple-scatterer cells as the main contributors to nonpersistence. It is striking how well the error-vs-resolution trends for the three real objects match the trends for the simulated object as shown in Fig. 25.

It should be noted that the error-vs-resolution trends for Slicy for the polarizations HV and VH as shown in Fig. 28 are not as flat as all of the other magnitude trends. This can be understood to be a result of the fact that scatterers are typically not bright in the HV and VH polarizations, so that noise becomes a more dominant factor. When compounded with the fact that at most angles Slicy has very little backscattering, it becomes clear why Slicy has such noisy yet flat trends for the HV and VH parameter spaces.

3. Measured persistence-vs-resolution trends

The persistence-vs-resolution trends of the three real scattering objects as shown in Figs. 33-38 are also consistent with those predicted and are again in striking agreement with the simulated trends shown in Fig. 26.
FIG. 33. Slicy's persistence-vs-resolution trends for the Euler parameters.

FIG. 34. Slicy's persistence-vs-resolution trends for the HH-VV parameters.
FIG. 35. Simulator's persistence-vs-resolution trends for the Euler parameters.

FIG. 36. Simulator's persistence-vs-resolution trends for the HH-VV parameters.
FIG. 37. T-72's persistence-vs-resolution trends for the Euler parameters.

FIG. 38. T-72's persistence-vs-resolution trends for the HH-VV parameters.
The angular Euler parameters all show the expected persistence-vs-resolution trends for each of the three objects according to Figs. 33, 35, and 37. As predicted, worse resolutions (larger pixel sizes) lead to more cells containing multiple scatterers and thus the overall azimuthal persistence decreases. The magnitude parameters, $m$, HH, HV, VH, and VH all show the predicted flat trends in Figs. 34, 36, and 38 indicating that they are not sensitive to multiple-scatterer cells.

The presence of multiple scatterers inside single pixels has been proven to be consistently one of the main causes of Euler accuracy degradation and Euler persistence degradation. This characterization leads to the conclusion that the overall degradation of Euler parameter accuracy can indeed be minimized by the weighting down of multiple-scatterer cells as described in Sec. D. of the Methodology.

**B. Effect of persistence optimization on ATR performance**

With the previous results having confirmed the role of multiple-scatterer pixels in decreasing Euler parameter accuracy, the effect of persistence optimization on ATR performance can now be investigated. The first set of results pertains to the controlled ATR tests where azimuth was known. The following section, Sec. C. of the Results, shows the performance results of the complete ATR algorithm.

The effect of persistence optimization on the controlled ATR performance of the angular Euler parameters is first presented in Figs. 39-46. Each trend-line in the probability density plots represents a single vehicle in the test suite compared to a single vehicle in the reference suite over all 360 possible azimuth angles. The plots are labeled by the tank code names, CI, CK/CL, AZ, as detailed in the Appendix.
FIG. 39. APD's of $\gamma$ images without persistence optimization.

FIG. 40. APD's of $\gamma$ images with persistence optimization.

FIG. 41. APD's of $\psi$ images without persistence optimization.

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FIG. 42. APD's of $\psi$ images with persistence optimization.

FIG. 43. APD's of $\tau$ images without persistence optimization.

FIG. 44. APD's of $\tau$ images with persistence optimization.
The angular Euler parameters without persistence optimization can be seen in Figs. 39, 41, 43, and 45 to give large curve overlaps between the dark, same-target trend-lines and the lighter, different-target trend-lines. This large area of overlap indicates that when treated separately, the unoptimized angular Euler parameters yield poor ATR performance. However, because the angular Euler parameters are independent properties of the same scattering object, they can still improve ATR performance when consolidated. The consolidation of the Euler parameters is discussed in a subsequent
section, Sec. C. of the Results.

It should be remembered that the suite of vehicles tested here represent the most challenging ATR task and that the Euler parameters would give much better ATR performance for the traditional task of vehicle differentiation.

When persistence optimization is used with the angular Euler parameters as shown in Figs. 40, 42, 44, and 46 there is small but significant ATR performance improvement. That there is some improvement confirms the role of multiple-scatterer cells in accuracy degradation, but that the improvements are so small indicates that the ATR performance is effected by other degradation phenomena.

The effect of persistence optimization on the magnitude parameters \( m, \) HH, HV, VH, and VV, is next considered in Figs. 47-56.

![FIG. 47. APD's of \( m \) images without persistence optimization.](image-url)
FIG. 48. APD's of $m$ images with persistence optimization

FIG. 49. APD's of HH images without persistence optimization.

FIG. 50. APD's of HH images with persistence optimization.
FIG. 51. APD's of HV images without persistence optimization.

FIG. 52. APD's of HV images with persistence optimization.

FIG. 53. APD's of VH images without persistence optimization.
FIG. 54. APD's of VH images with persistence optimization.

FIG. 55. APD's of VV images without persistence optimization.

FIG. 56. APD's of VV images with persistence optimization.
It can be seen in Figs. 47, 49, 51, 53, and 55 that the magnitude parameters without persistence optimization already show smaller curve overlap and thus better ATR performance than the angular Euler parameters. When persistence optimization is applied, the magnitude parameters (see Figs. 48, 50, 52, 54, and 56) show significant ATR performance improvement. This is in contradiction with the prediction made by the multiple-scatterer-cell hypothesis that persistence optimization should degrade magnitude parameter accuracy. This may be understood by the concept that the nonpersistence may be resulting from other sources of nonpersistence as well as from the multiple-scatterer-cell phenomenon. Apparently, persistence optimization affects some other nonpersistence phenomenon in addition to multiple-scatterer-cell nonpersistence, and the magnitude parameters see unexpected improvement. It should be noted that when the ATR tests were performed with the full ATR algorithm, as presented in Sec. C. of the Results, persistence optimization applied to the magnitude parameters did not consistently lead to this unexpected improvement. Thus the unexpected results are dependent on test environment.

Out of all of the parameters considered, the one that shows smallest curve overlap is the Euler parameter maximum magnitude $m$ with persistence optimization turned on (see Fig. 48). The parameter $m$ lead to better ATR performance than any of the traditional magnitude parameter spaces. In fact, for this particular suite of vehicles in the high-fidelity compact-range environment and for the controlled tests, the persistent-optimized parameter $m$ in Fig. 48 shows complete curve separation and thus perfect target recognition.

It should be remembered that the Euler parameters have been treated separately
here for analysis purposes, and that they become more effective in ATR when consolidated as discussed at the end of Sec. C. of the Results.

C. Performance of the full ATR algorithm

The same tests performed in the controlled-variable environment were repeated in the full environment using the complete ATR algorithm described in the Methodology section. The complete ATR requires no previous or external knowledge about the unknown vehicle and thus the performance results are a better representation of what is to be expected in an actual, field scenario. To ensure statistical significance for the full ATR results, thousands of test images were used of the different test vehicles at different look angles. With the use of a large number of test vehicles and azimuth angles, presentation of the results in the form of the probability density distribution used above is too involved to be visually meaningful. Instead, the results of the full ATR performance test are presented in the form of Receiver Operating Characteristic (ROC) curves to give better summaries of performance while diminishing focus on individual comparisons.

When viewing ROC curves, it must be remembered that absolute curve position is highly dependent on the set of test objects and reference objects used. To ensure that the ROC curves best represent the performance of the full ATR algorithm under one of the most difficult tests, only the spatially similar tanks described in the Methodology Section were included in the test set and reference library. Inclusion of other vehicles, such as the M1 tank or trucks, in the test suite or library suite would only improve the ATR performance results, pushing the ROC curve far up into the left-top corner of the plot, and obscure the effects of the Euler parameters and persistence optimization. These other
vehicles were therefore omitted from the library for all of the tests.

The performance of the full ATR algorithm without persistence optimization is first presented in Figs. 57 and 58, where each parameter is treated separately. Each ROC curve represents the average target recognition performance of the full ATR algorithm when the four test tanks in the test suite, CI-1, CK-1, CL-1, and CL-2 were tested against the three tanks of the library, CI-2, CK-2, and AZ at the 360 different possible azimuthal angles. Further information regarding the meaning of the tank codes can be found in the Appendix. Similar to the results found in the controlled-variable test environment in Sec. B of the Results, the magnitude parameters in the complete ATR test lead to better ATR

![ROC curves without persistence optimization](image-url)

FIG. 57. ROC curves without persistence optimization.
performance than the Euler angular parameters when treated separately. Fig. 57 gives a sense of how much better the magnitude parameters perform.

In order to better visualize the ATR performance of the unoptimized magnitude parameters, the scales can be adjusted on Fig. 57 to zoom in on the top left corner. The resulting plot in Fig. 58 makes it evident that the Euler magnitude parameter $m$ leads to the best ATR performance, in agreement with previous results.

Next, the effect of persistence optimization on the performance of the full ATR algorithm is presented when the parameters are treated separately. The Euler angular parameters see significant ATR performance improvement when persistence optimization...
is applied, as shown in Fig. 59. The orientation angle parameter $\psi$ and the polarizability parameter $\gamma$ experience the most dramatic improvement. These results confirm the predictions made by the multiple-scatterer-cell hypothesis.

In contrast, the magnitude parameters experience no additional effect when persistence optimization is applied. As shown in Fig. 60, the magnitude parameters in general experience no improvement in ATR performance when persistence optimization
FIG. 60. ROC curves of persistence-optimized magnitude parameters.
is used, and possibly even minor degradation. These findings confirm the predictions made by the multiple-scatterer-cell hypothesis. The possible slight amount of degradation is in agreement with the magnitude parameter persistence-vs-resolution trends. It should be noted that the ROC curves of $m$, HH, and VV in Fig. 60 are plotted on a zoomed-in scale in order to make visual comparisons possible, whereas the ROC curves of HV and VH in Fig. 60 are plotted on the full-scale because their lower performance requires a broader scale. It should also be noted that the traditional parameter VV does show minor improved ATR performance when persistence optimization is applied, in disagreement with all of the other magnitude parameters.

It should be remembered that each ROC curve represents the total ATR performance of the parameter concerned when all of the vehicles in the test suite are tested at every possible angle, and thus each curve has broader significance than any individual comparison.

Lastly, a consolidated score was established that leads to the best ATR performance. Even though the angular Euler parameters show worse target recognition than the traditional magnitude parameters when treated separately, they can still contribute to improved ATR performance when consolidated. This arises from the fact that a set of Euler parameters specify independent properties of the same scattering object. To achieve optimal performance, the consolidation method should combine the persistence-optimized Euler angular parameters $\gamma$, $\tau$, $\psi$, $\nu$ with the unoptimized Euler magnitude parameter $m$. The traditional parameters HH, HV, VH, VV are not independent of the Euler parameters, and therefore would contribute nothing if included in the consolidation.
It was found that the consolidation method that leads to the best ATR performance involves combining the optimized Euler parameters into a consolidated score $s$ according to Eq. (34).

$$s = \sqrt{(s_m - s_m)^2 + (s_r - s_r)^2 + (s_y - s_y)^2 + (s_{\psi} - s_{\psi})^2 + (s_{\phi} - s_{\phi})^2}$$  

(34)

Here $s_m$, for example, is the Average Percent Difference score between the unknown vehicle and a reference vehicle when imaged in Euler $m$ parameter space. The centering terms $s_{m0}$, $s_{\phi0}$, $s_{\theta0}$, $s_{\psi0}$, $s_{\phi0}$ in Eq. (34) are necessary because the Euler parameters operate on different scales. These centering terms are found using an algorithm that adjusts the terms until performance is optimized for a training data set. Once found, the same centering terms are used for all subsequent tests. The data set used to train the consolidation equation $s$ for the optimal centering terms is necessarily different than the data sets used to test performance.

Conceptually, the five independent Euler parameter APD scores can be thought of as defining a five-dimensional space, where any image comparison is represented by a point in the five-dimensional space. The consolidated score $s$ in Eq. (34) then represents the distance in this five-dimensional space between a given comparison and some fixed reference center. Several other consolidation schemes were attempted, but they were found to give similar results, and often worse results than Eq. (34).

The final ATR performance results are presented in Fig. 61. When treated separately, the Euler parameter with the best target recognition is the maximum magnitude $m$, which does better than the traditional HH imaging space. However, the best ATR performance was achieved by using the consolidated, optimized Euler score $s$. 

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While the ROC curve midpoint, shown as a box in Fig. 61, for the ATR algorithm using the traditional HH parameter space shows 98.9% true-positive identification rate, the Euler parameter $m$ improves the performance curve midpoint to 99.4% true-positives, and the consolidated Euler score $s$ improves performance to a 99.7% true-positive identification rate.

**FIG. 61.** Zoomed-in ROC curves of the best ATR configurations.
IV. DISCUSSIONS

The error-vs-resolution trends as well as the persistence-vs-resolution trends all confirm that multiple-scatterer cells degrade the accuracy of angular Euler parameters but not the accuracy of the magnitude parameters. These results indicated that the angular parameters, $\gamma$, $\psi$, $\tau$, and $\nu$ should have seen great accuracy improvement and thus corresponding ATR performance improvement when persistence optimization was applied. The multiple-scatterer cell results also indicated that the magnitude parameters, HH, HV, VH, VV, and $m$ should have seen no improvement in ATR performance when persistence optimized, and even slight performance degradation.

The controlled-variable and full-ATR tests confirmed the predictions of the multiple-scatterer-cell hypothesis. However, the magnitude-parameter ATR configurations did not always show degraded performance as predicted when persistence optimization was applied. The few cases that were found where the persistence-optimized magnitude parameters gave improved performance indicate that there must be some other phenomenon, in addition to multiple-scatterer cells, that degrades persistence. Also noteworthy is that when persistence optimization was applied to the Euler angular parameters, their ATR performance improved but still fell quite short of the magnitude parameter's ATR performance as shown in Fig. 59. This fact indicates that there is some other phenomenon degrading target recognition in addition to multiple-scatterer cells.

Despite the persistence complexities and the unexpected results, the multiple-scatterer-cell hypothesis still holds true enough generally that the best performing full-
ATR algorithm was found to be the one using the persistence-optimized Euler parameters consolidated into a single score as shown in Fig. 61.
V. CONCLUSIONS

Although the results indicate a more complex interplay of parameter degradation phenomena than originally supposed, several conclusions can be made concerning this research: First, the ambiguities inherent to the Euler transformation can be completely removed and simple objects can be imaged in intuitive, phenomenological parameter spaces.

Second, the presence of multiple scattering centers in image cells was shown to be a major cause of azimuthal nonpersistence and ATR performance degradation for the Euler parameters in addition to natural nonpersistence. The impact of multiple-scatterer cells on target recognition was shown to be minimized by optimizing the Euler parameters according to their persistence.

Finally and most importantly, a new, full ATR algorithm has been developed using a novel azimuth-finding method, novel imaging techniques, and using the persistence-optimized Euler parameters consolidated to a score $s$, which is more successful at automatic target recognition than the traditional parameter approaches. Although the improvement in target recognition that is gained by using the Euler parameters and persistence optimization is often small, it is consistently significant. The best target recognition configuration was found to be when the full ATR algorithm uses the persistence-optimized Euler parameters, consolidated to a score $s$. 

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VI. RECOMMENDATIONS

Several avenues of future research recommend themselves in order to corroborate and extend this research. Repeating the full ATR tests with different suites of test vehicles would further confirm the results and better isolate the role of vehicle shape and complexity in the degradation phenomena. Also, novel techniques could be developed to investigate other causes of non-perfect ATR performance and, if possible, minimize the causes.
VII. LITERATURE CITED


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ADDITIONAL REFERENCES USED BUT NOT CITED


APPENDIX

The target scattering datasets used in this research to test ATR performance were obtained from a previous study on target variability (2). To better understand the datasets involved and the meaning of the codes assigned to them, an extract of the variability study is reproduced verbatim in Table 2. The abbreviated codes used in this research were as follows: CL-1 designates 01B050KBBA.CLD, CL-2 designates 01B050KBAA.CLD, CI-1 designates 01A050KBAA.CID, CI-2 designates 00L050KBAA.CID, CK-1 designates 01C050KBAA.CKD, CK-2 designates 01B050KBAA.CKD, and AZ designates 01B050KBBA.AZD.

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<th>Turntable</th>
<th>Drums</th>
<th>Turret</th>
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|          | #940    | 5     | free space| none  | 9      |       | 01B050KBCA.CKD        |

| T-72M1#2 | #940    | 5     | free space| none  | 0      |       | 01B050KBCA.CLD        |
|          | #940    | 5     | free space| none  | 36     |       | 01B050KBDA.CLD        |
|          | #940    | 5     | free space| none  | 9      | yes   | 01B050KBAA.CLD        |
|          | #940    | 5     | free space| none  | 9      | yes   | 01B050KBBA.CLD        |

| T-80UD   | generic | 5     | free space| none  | 9      | yes   | 01B050KBBA.AZD        |
|          | generic | 15    | free space| none  | 9      |       | 01B150KBAA.AZD        |

TABLE 2. Datasets obtained in a previous study (2).
BIOGRAPHICAL SKETCH OF AUTHOR

Christopher S. Baird obtained his primary education in the Belmont Public School system. He attended Brigham Young University (BYU) in 1996, served a full-time mission in Germany for the Church of Jesus Christ of Latter-Day Saints from 1997-1999, and studied again at BYU from 1999-2001. He was awarded the Bachelor's in Science degree from BYU in physics in 2001 with a minor in mathematics and an emphasis on computer science.

After marrying Ellen Shumway in the Spring of 2001, Christopher S. Baird worked as a professional-level web developer and programmer for ZServe Corporation located in Provo, Utah. He commenced graduate studies in 2002 at the University of Massachusetts Lowell (UMass Lowell), and pursued research in electromagnetic scattering at the Submillimeter-Wave Technology Laboratory under Dr. Robert H. Giles. In 2007, he received the Doctor of Philosophy degree in physics from UMass Lowell. Mr. Baird currently has three children and is pursuing a career in radar scattering.